Higher order terms in the kernel

Non-locality in space

- For turbulence with energy-carrying wavenumber k_f, a good approximation to the kernel is of the form $\exp(-k_\mathrm{f}|\boldsymbol{x}-\boldsymbol{x}'|)$; see Brandenburg et al. (2008).
- This can be modelled as diffusion term in an evolution equation for the e.m.f., i.e., $\partial\overline{\boldsymbol{\mathcal{E}}}/\partial t = ... + \eta_\mathcal{E}\nabla^2\overline{\boldsymbol{\mathcal{E}}}$; see Rheinhardt & Brandenburg (2012).
- Nonlocality in space can lead to dynamo action via negative turbulent diffusivity. An example is the Roberts IV flow; see Devlen et al. (2013).

Non-locality in time

- For turbulence with rms velocity u_rms, a good approximation to the kernel is of the form $\Theta(t-t')\exp[-u_\mathrm{rms}k_\mathrm{f}(t-t')]$; see Hubbard & Brandenburg (2009).
- As a practical approximation, this can be modelled as damping term in a time-dependent e.m.f., i.e., $\partial\overline{\boldsymbol{\mathcal{E}}}/\partial t = ... - u_\mathrm{rms}k_\mathrm{f}\overline{\boldsymbol{\mathcal{E}}}$; see Brandenburg et al. (2004).
- Nonlocality in time can lead to dynamo action via a time delay. Examples include the Roberts II and III flows; see Rheinhardt et al. (2014).

Higher order terms in solving for the fluctuations

Here some concrete examples where SOCA fails:

- A transport term emerges at sixth order in the Galloway–Proctor flow; see Rädler & Brandenburg (2009).
- A negative contribution to turbulent diffusivity emerges at fourth order in helical turbulence; see Brandenburg et al. (2017).
- A small negative contribution to turbulent passive scalar diffusivity emerges at second order in compressible turbulence; see Rädler et al. (2011).

Analogy between hydro and MHD forcings

Cross-helically forced flows (e.g., with the Roberts I flow) exhibit new similarities with the kinetically forced case; see Rheinhardt & Brandenburg (2010).

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