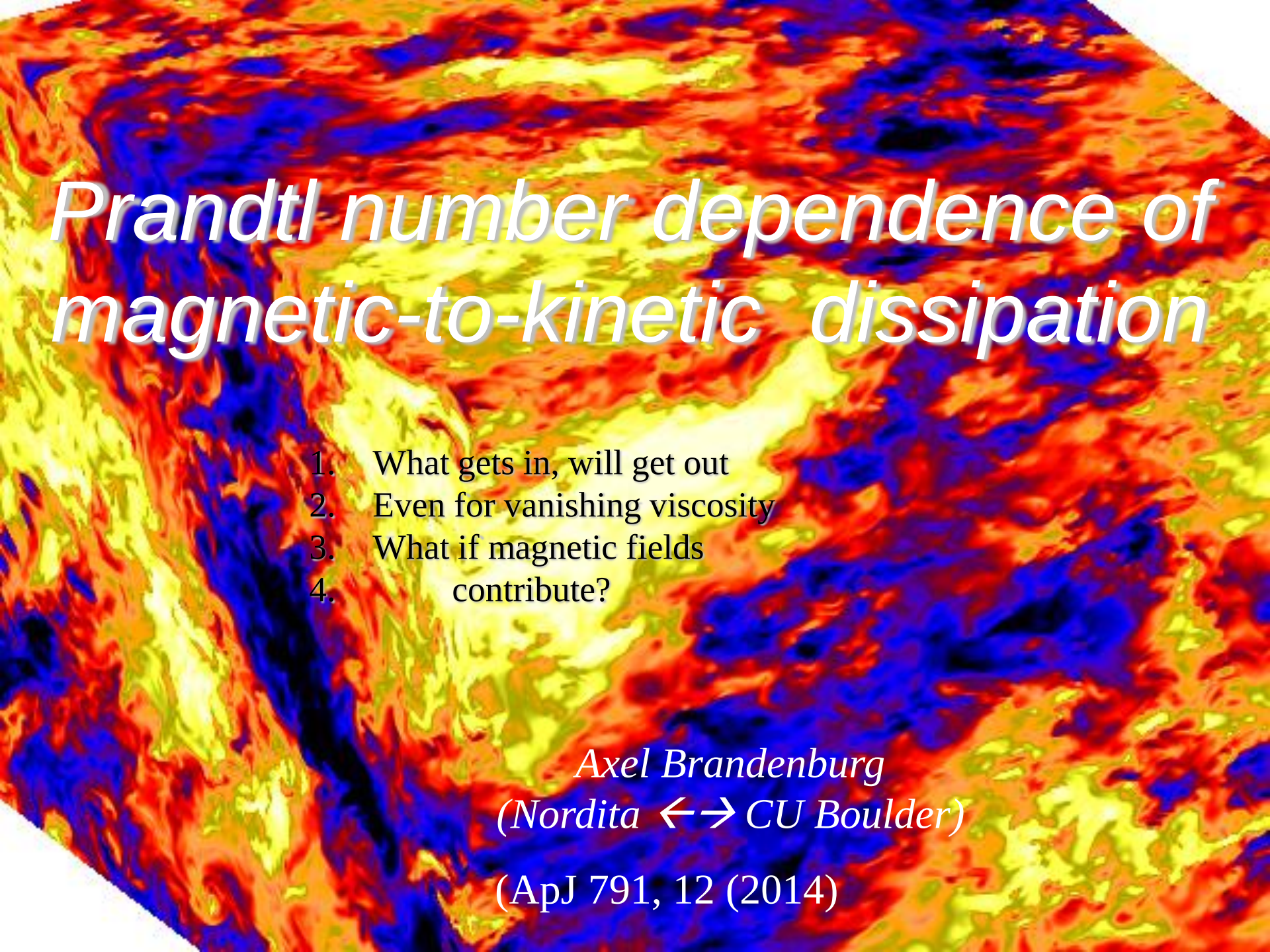


Prandtl number dependence of magnetic-to-kinetic dissipation

1. What gets in, will get out
2. Even for vanishing viscosity
3. What if magnetic fields
4. contribute?

Axel Brandenburg
(Nordita ←→ CU Boulder)

(ApJ 791, 12 (2014)

Finite dissipation at vanishing viscosity

$$\frac{d}{dt}\left\langle \rho \mathbf{u}^2/2 \right\rangle = \left\langle \rho \mathbf{u}\cdot\mathbf{f} \right\rangle - \left\langle \rho \nu \boldsymbol{\omega}^2 \right\rangle$$

$$\left\langle 2\rho\nu \mathbf{S}^2 \right\rangle$$

if $\nu \to 0$ then $\omega^2 \to$ infty

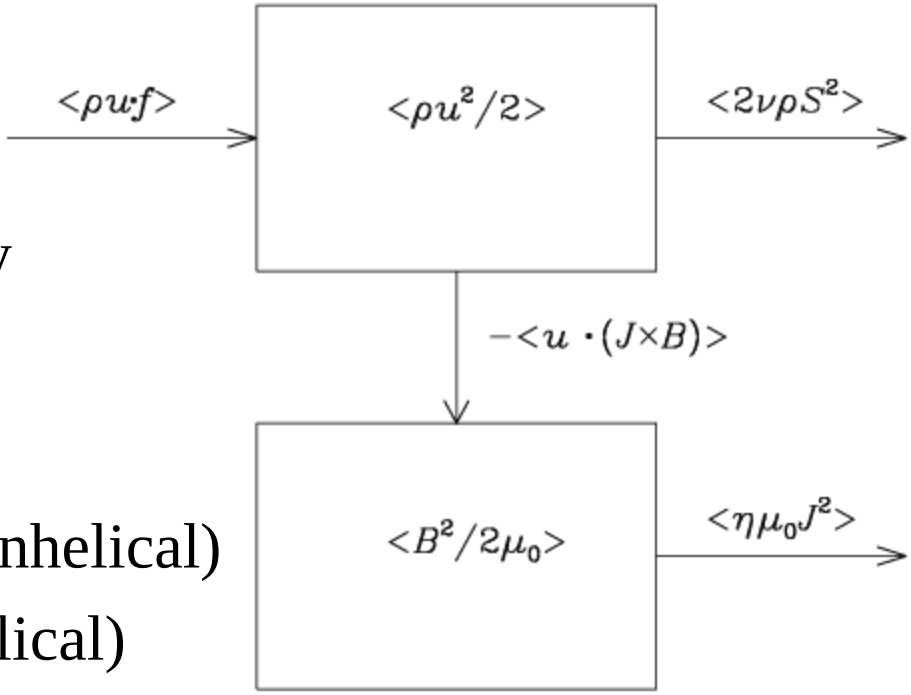
Traceless rate-of-strain tensor $S_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) - \frac{1}{3}\delta_{ij}u_{k,k}$

How is this modified by magnetic fields?

- Smaller η, more J, same dissipation
- Or: dynamo stronger, more dissipation
- Or: less dissipation ←

Couple to B-field

- $\langle \boldsymbol{u}.(\boldsymbol{J}\times\boldsymbol{B})\rangle$ taps energy
  - Dynamo if negative
  - Self-excited if instability
  - Requires 3-D $\boldsymbol{B}$-field
- Isotropic turbulence
  - Small-scale dynamo (nonhelical)
  - Large-scale dynamo (helical)

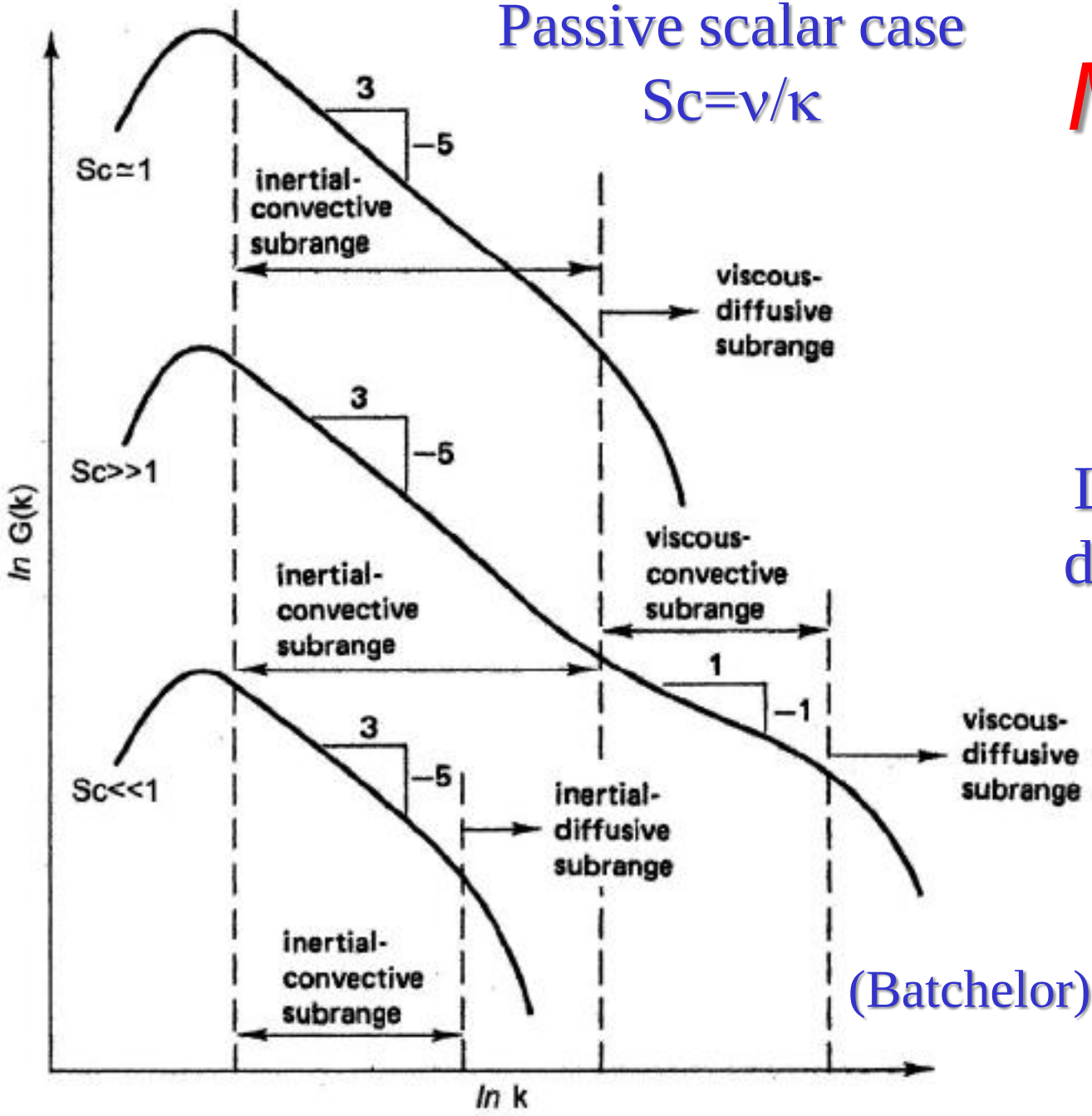


$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \rho \boldsymbol{u}^2/2\rangle = \langle p\boldsymbol{\nabla}\cdot\boldsymbol{u}\rangle + \langle \boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})\rangle + \langle \rho\boldsymbol{u}\cdot\boldsymbol{f}\rangle - \langle 2\rho\nu\mathbf{S}^2\rangle, \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \boldsymbol{B}^2/2\mu_0\rangle = -\langle \boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})\rangle - \langle \eta\mu_0\boldsymbol{J}^2\rangle, \tag{7}$$

Passive scalar case

$Sc = \nu/\kappa$



(Batchelor)

Motivation of LES modeling

Location of cutoff does not matter for inertial range

Order of limits

$\nu \rightarrow 0$

$\eta \rightarrow 0$

unimportant

Coupling to B-field

- Magnetic dissipation depends on work term
  - Independent of microphysics?
  - Basic assumption of LES and iLES
  - e.g., hyperdiffusion (Galsgaard & Nordlund 1996)
- Not confirmed by DNS
  - Ratio scales with ν/η
  - Either ν or η fixed
  - What if replace Spitzer by Hall?

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Inverse cascades and α effect at a low magnetic Prandtl number

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(Received 13 February 2007; revised manuscript received 25 June 2007; published 24 August 2007

Dynamo action in a fully helical Arn'old-Beltrami-Childress flow is studied using both direct num simulations and subgrid modeling. Sufficient scale separation is given in order to allow for large-scale netic energy buildup. Growth of magnetic energy obtains down to a magnetic Prandtl number $P_M=R$ close to 0.005, where R_V and R_M are the kinetic and magnetic Reynolds numbers. The critical mag Reynolds number for dynamo action R_M^c seems to saturate at values close to 20. Detailed studies c dependence of the amplitude of the saturated magnetic energy with P_M are presented. When P_M is decre numerical experiments are conducted with either R_V or R_M kept constant. In the former case, the rat magnetic to kinetic energy saturates to a value slightly below unity as P_M decreases. Examination of e spectra and structures in real space reveals that quenching of the velocity by a large-scale magnetic field place, with an inverse cascade of magnetic helicity and a force-free field at large scale in the saturated re

DOI: 10.1103/PhysRevE.76.026316 PACS number(s): 47.65.−d, 47.27.E−, 91.25.Cw, 95.3

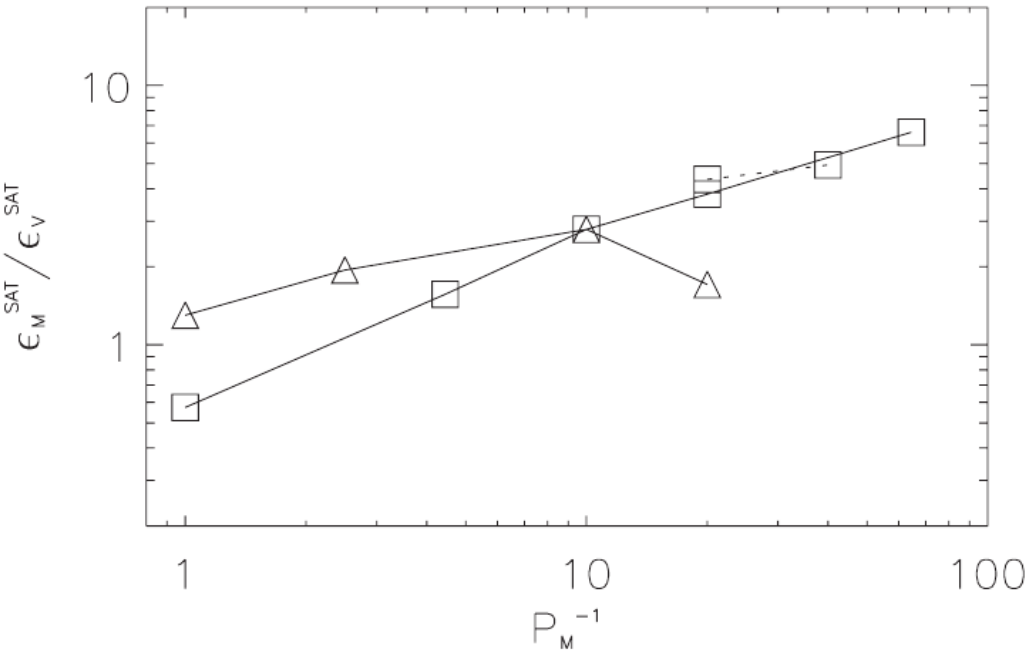


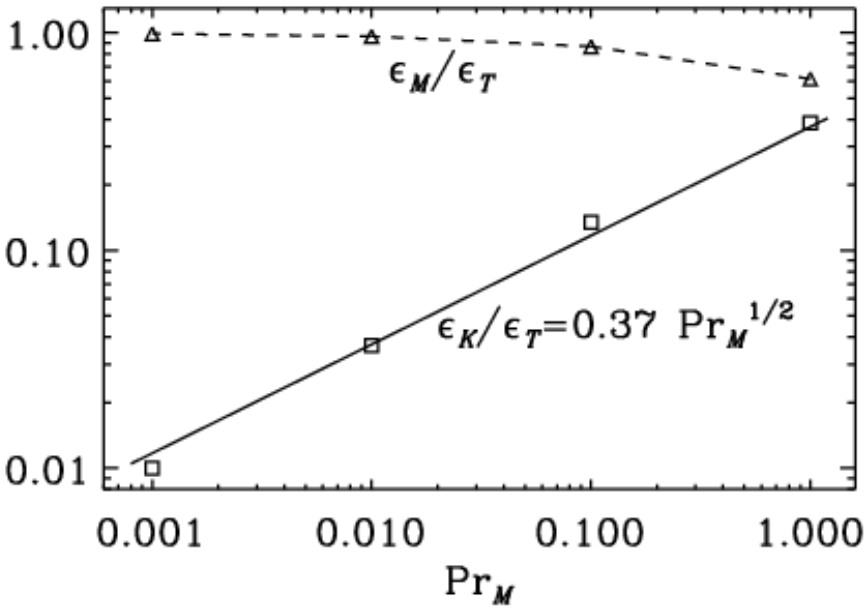
Figure 8 shows the ratio of the magnetic energy dissipation rate ϵ_M to the kinetic energy dissipation rate ϵ_V in the saturated state for the same runs as in Fig. 7. At constant R_V, for small values of P_M, a drop in the ratio is observed as the value of R_M gets closer to the threshold. On the other hand, at constant R_M, more and more energy is dissipated by Ohmic dissipation as P_M is decreased.

ν smaller, $\nu\omega^2$ smaller, ηJ^2 larger

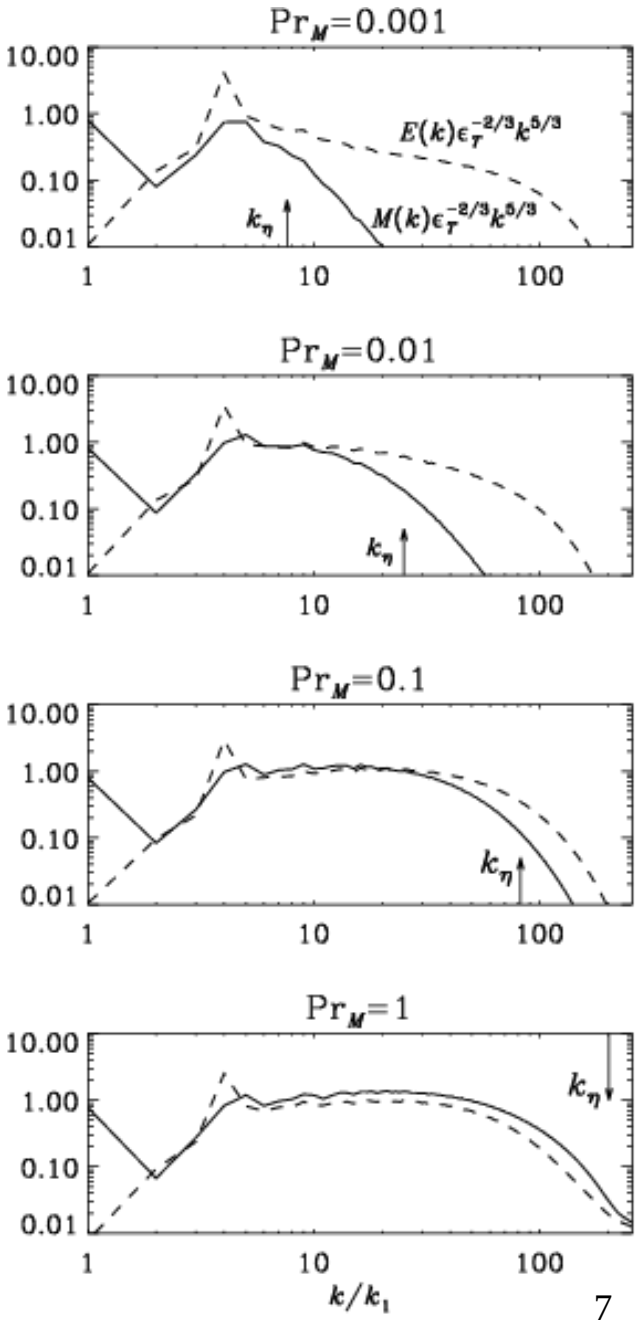
Numerically difficult?

- **Energy dissipation via Joule**
  - **Viscous dissipation weak**
- **Can increase Re *substantially*!**

$Pr_M = \nu/\eta$



Brandenburg (2009, ApJ)



Isothermal MHD

$$\frac{D\ln\rho}{Dt} = -\boldsymbol{\nabla}\cdot\boldsymbol{u},$$

$$\frac{D\boldsymbol{u}}{Dt} = -c_{\rm s}^2\boldsymbol{\nabla}\ln\rho - 2\boldsymbol{\Omega}\times\boldsymbol{u} + \boldsymbol{f} + \rho^{-1}\left[\boldsymbol{J}\times\boldsymbol{B} + \boldsymbol{\nabla}\cdot(2\nu\rho\mathbf{S})\right],$$

$$\frac{\partial\boldsymbol{A}}{\partial t} = \boldsymbol{u}\times\boldsymbol{B} - \eta\mu_0\boldsymbol{J},$$

Forcing: helical or nonhelical

where $\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{u}\cdot\boldsymbol{\nabla}$ is the advective derivative, $\boldsymbol{u}$ is the velocity, $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$ is the magnetic field, $\boldsymbol{A}$ is the magnetic vector potential, $\boldsymbol{J} = \boldsymbol{\nabla}\times\boldsymbol{B}/\mu_0$ is the current density, μ_0 is the vacuum permeability, and

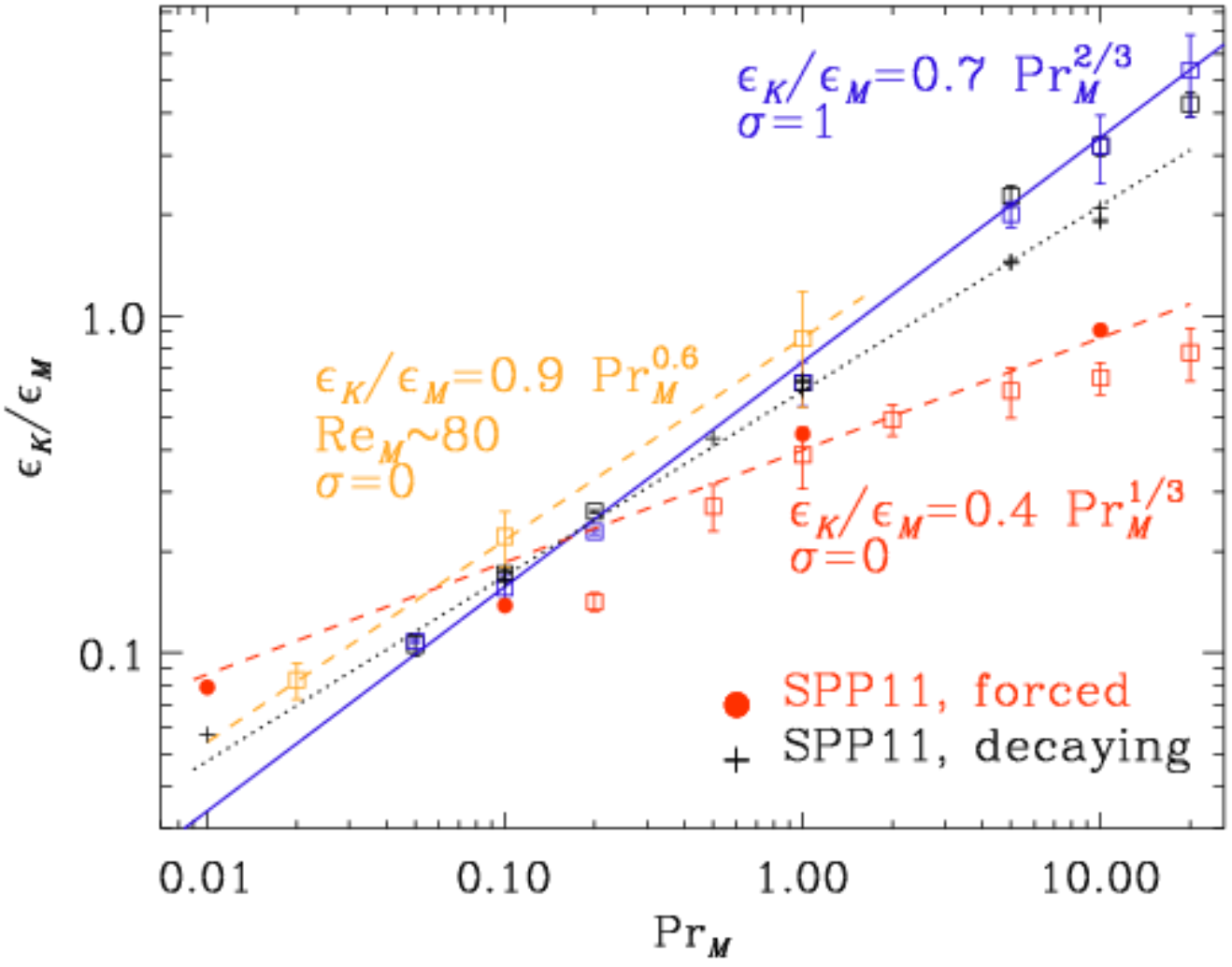
$$\mathsf{S}_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}) - \tfrac{1}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{u} \qquad (4)$$

is the traceless rate of strain tensor. It is useful to note that

$$\rho^{-1}\boldsymbol{\nabla}\cdot(2\rho\mathbf{S}) = \tfrac{4}{3}\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\boldsymbol{u} - \boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{u} + \mathbf{S}\cdot\boldsymbol{\nabla}\ln\rho, \quad (5)$$

where we call attention to the presence of the $4/3$ factor that

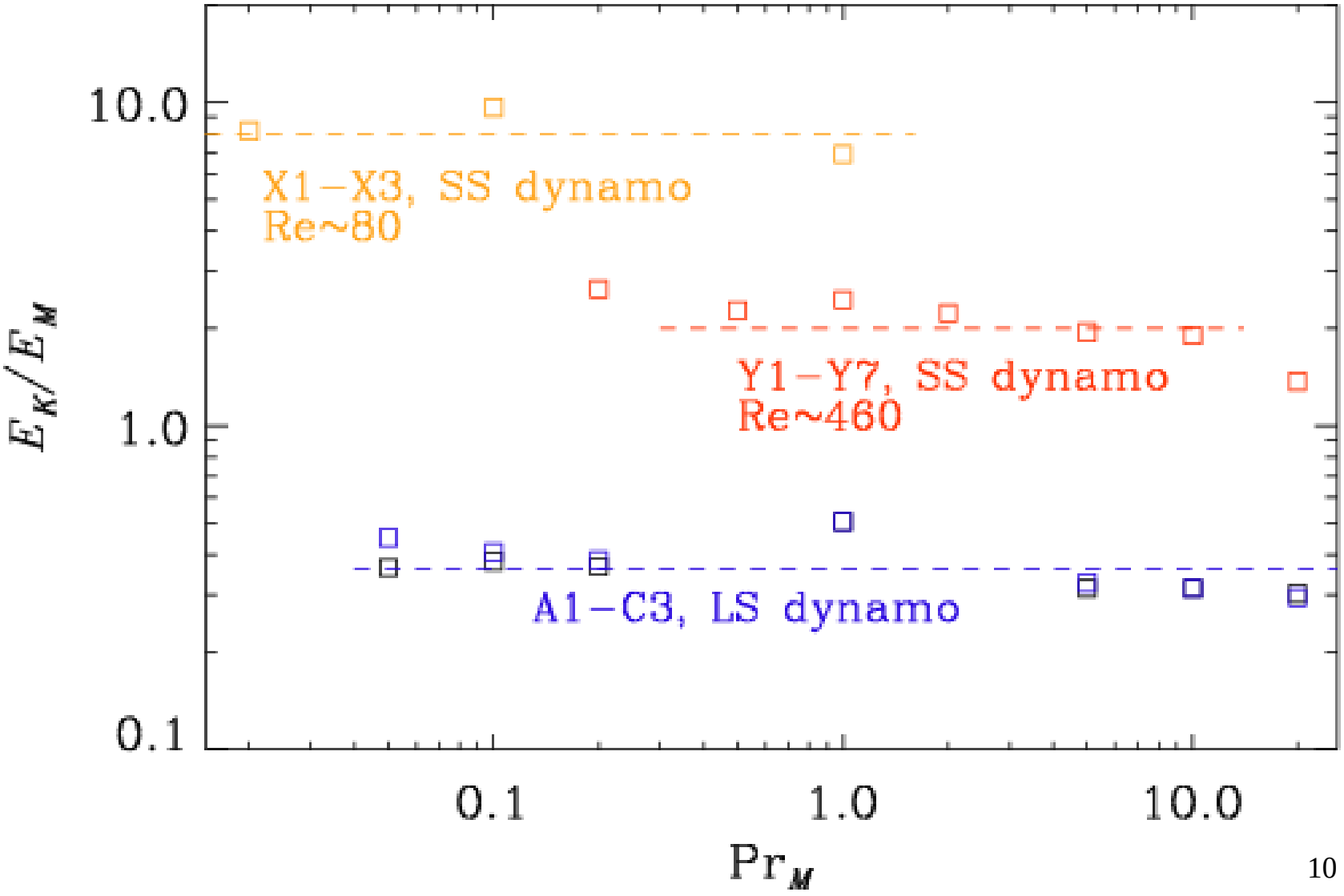
Pr_M dependence of dissipation ratio



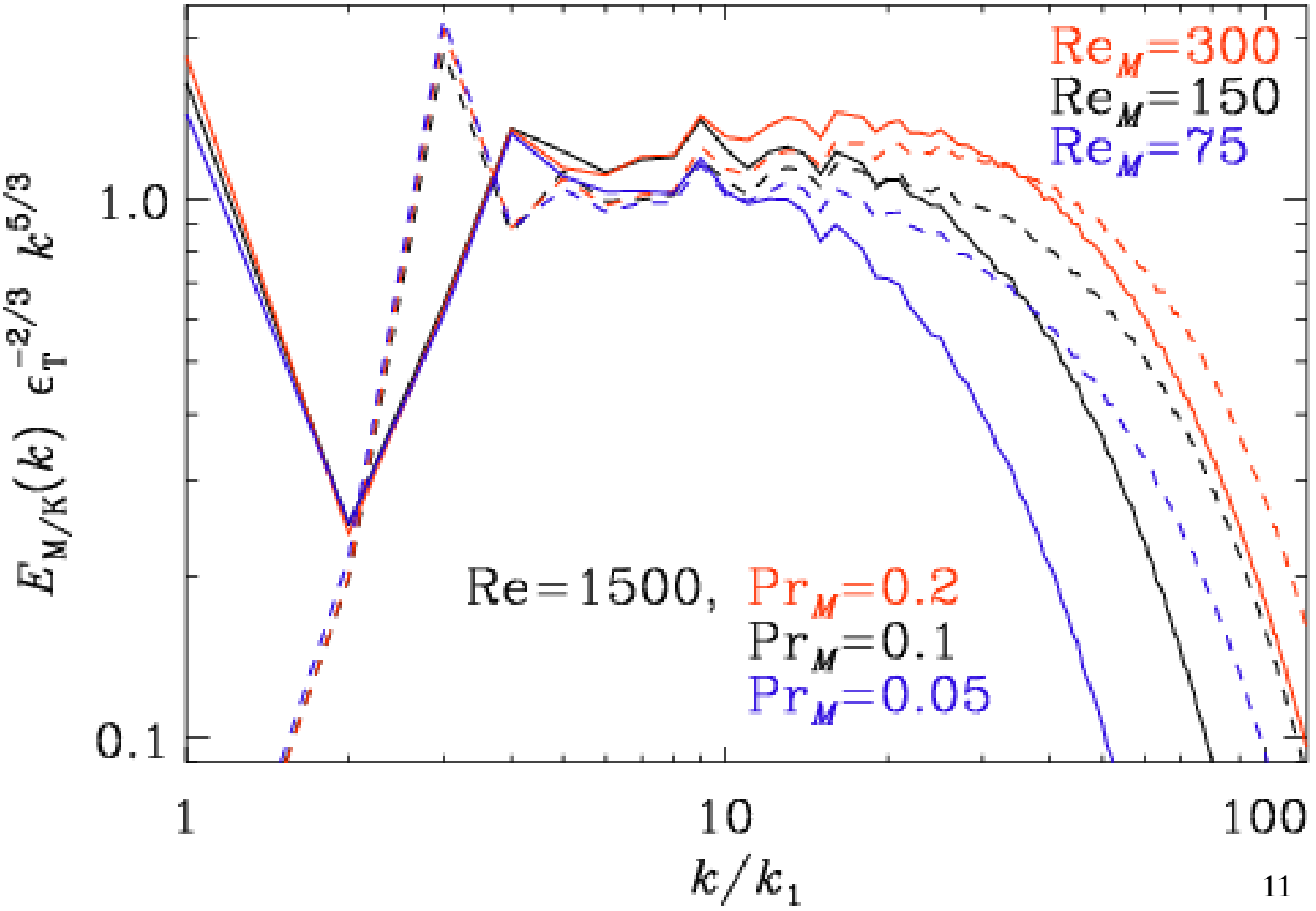
Brandenburg (2014, ApJ, 791, 12)

SPP = Sahoo, Perlekar, Pandit (2011)

Energy ratio nearly unchanged



Inertial range → compensated spectra



2-D MHD (Tran et al, JFM 2013)

this linear behaviour turns out to be fully accessible to numerical simulations. Indeed, the results reported in §4 show a nearly linear decrease of $\nu\|\omega\|^2$ with ν even for moderately small Pm. Second, we have

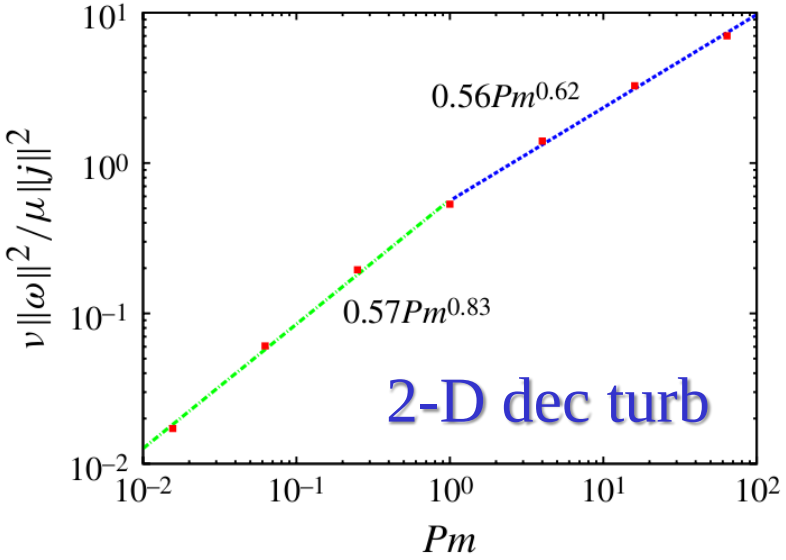
$$\int_0^t \|\nabla j\|^2 \, \mathrm{d}\tau < \infty. \tag{3.10}$$

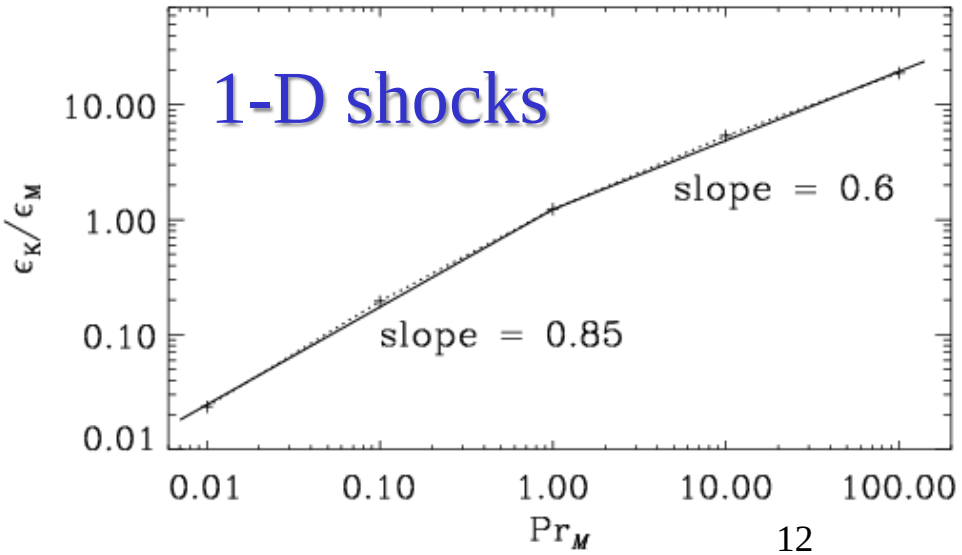
In conclusion, we see that for solution regularity beyond $t = T$ of the 2D MHD system with $Pm = \infty$, it is sufficient to require either

$$\frac{\|j\|_\infty}{\|j\|} \leqslant C(T-t)^{-\alpha} \quad \text{or} \quad \frac{\|\nabla u\|_\infty}{\|\omega\|} \leqslant C(T-t)^{-\alpha} \tag{3.39}$$

for $\alpha < 1/2$.

η smaller,
ν/η larger,
$\mathbf{J}^2$ finite,
$\eta\mathbf{J}^2 \rightarrow 0$
$\nu\omega^2/\eta\mathbf{J}^2$
keeps incr.





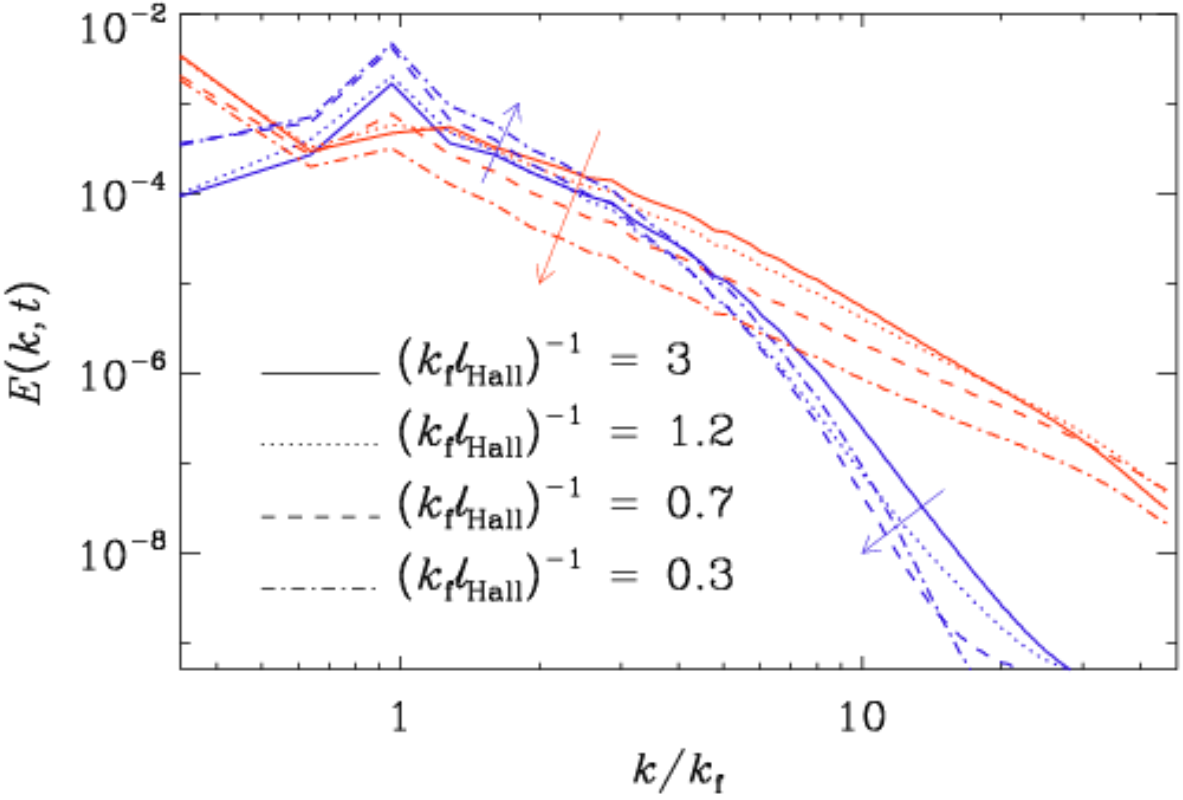
Brandenburg (2014, ApJ, 791, 12)

Hall MHD

- How does it affect dissipation ratio?
- Does it “replace” ohmic diffusion somehow
- Does it affect the dynamo
  - Backscatter from magnetic to kinetic energy (Mininni, Alexakis, Pouquet 2006)
  - Nature of MHD Alfven waves changed

Hall MHD simulations

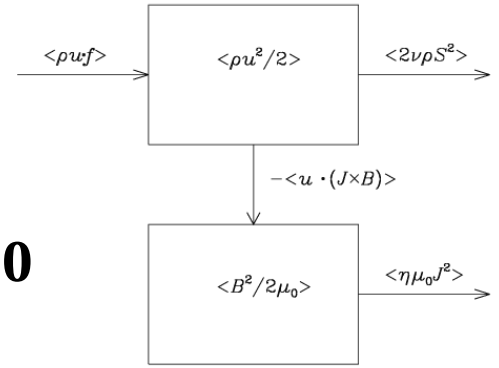
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{u} \times \mathbf{B} - \ell_{\mathrm{Hall}} \mathbf{J} \times \mathbf{B} - \eta \mu_0 \mathbf{J} \right)$$



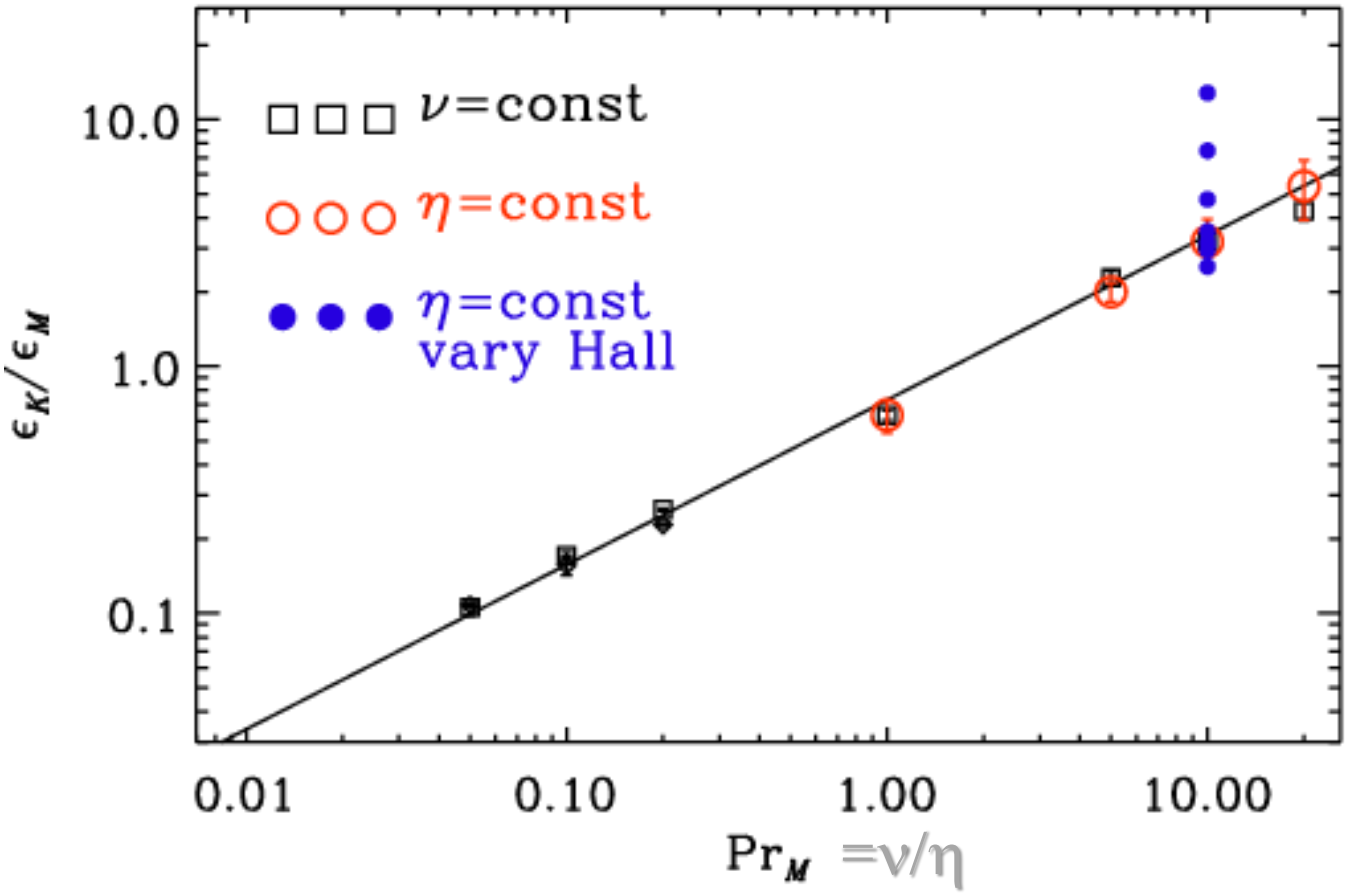
288^3 resol.

Makes dynamo weaker → less magnetic dissipation

Hall effect

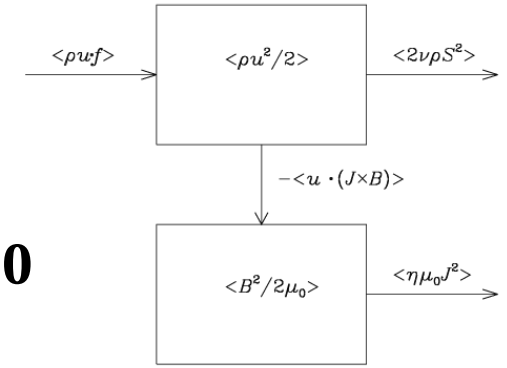


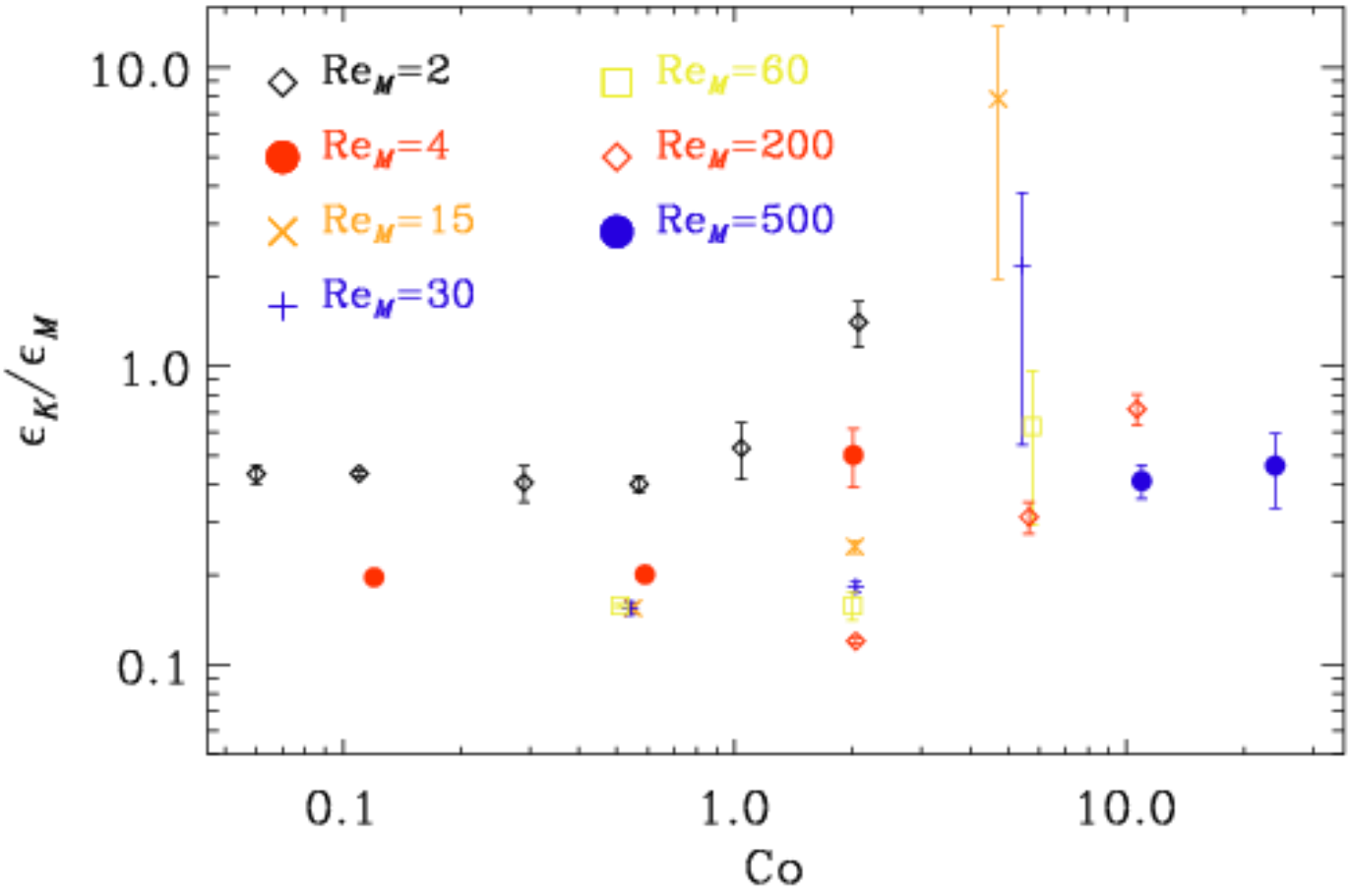
Makes dynamo weaker $\mathbf{J}\cdot\left(\ell_{\mathrm{Hall}}\mathbf{J}\times\mathbf{B}\right)=\mathbf{0}$



Effect of rotation

No effect if supercritical $\mathbf{u}\cdot\left(2\boldsymbol{\Omega}\times\mathbf{u}\right)=0$





Conclusions

- Dissipation ratio scales with Pr_M
  - Both for $Pr_M > 1$ and < 1
  - SS dynamo scaling shallower (nonuniversal)
- Qualitatively reproduced with MHD shocks
- <**u.**(**JxB**)> determined by microphysics!?
  - Hall does affect dynamo, if $1/\ell_{Hall}$ subinertial
  - Questions about LES or iLES