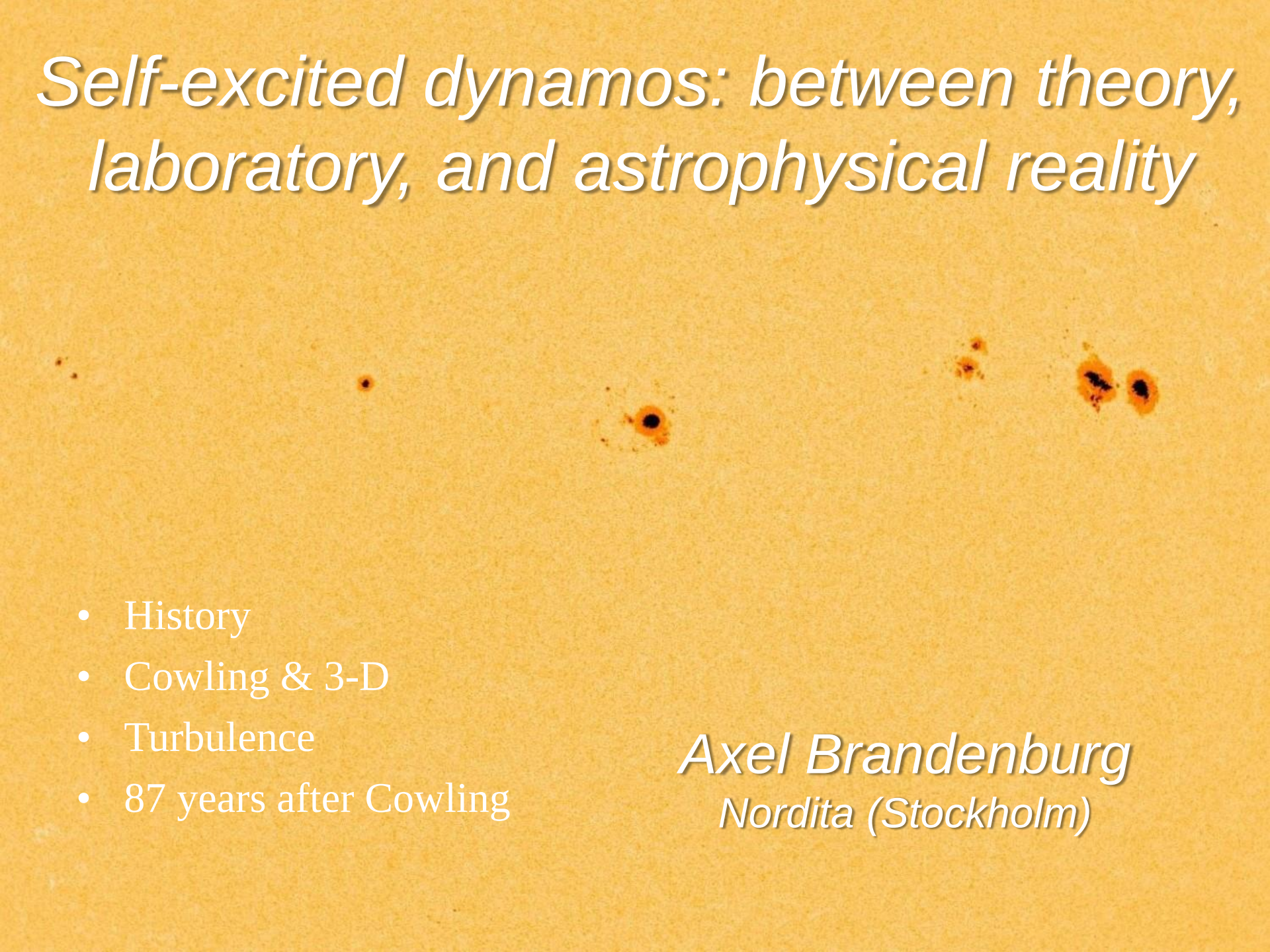
Self-excited dynamos: between theory, laboratory, and astrophysical reality



- History
- Cowling & 3-D
- Turbulence
- 87 years after Cowling

Axel Brandenburg
Nordita (Stockholm)

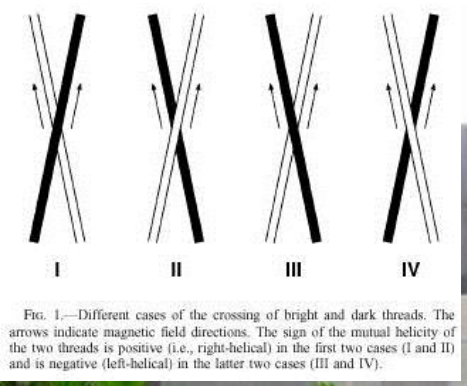


FIG. 1.—Different cases of the crossing of bright and dark threads. The arrows indicate magnetic field directions. The sign of the mutual helicity of the two threads is positive (i.e., right-helical) in the first two cases (I and II) and is negative (left-helical) in the latter two cases (III and IV).

PHYSICAL REVIEW E **84**, 016406 (2011)

Decay of helical and nonhelical magnetic knots

Simon Candelaresi and Axel Brandenburg
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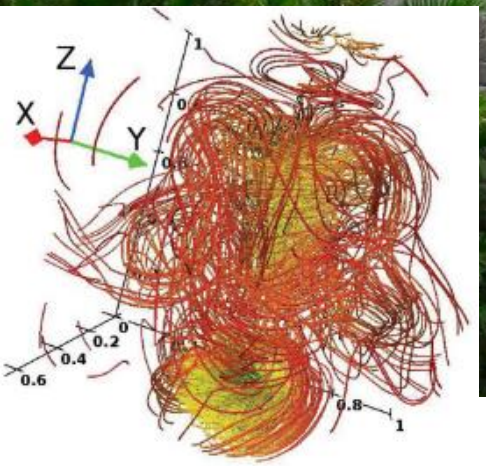
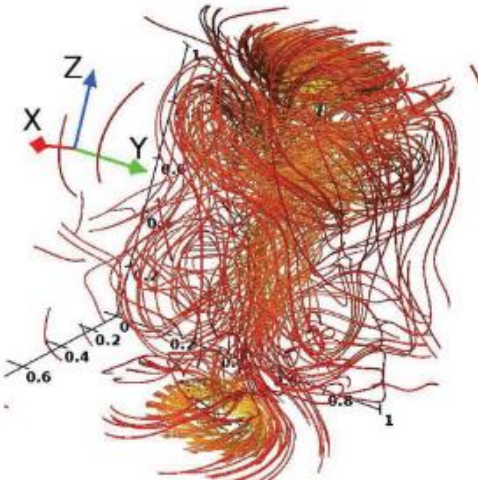
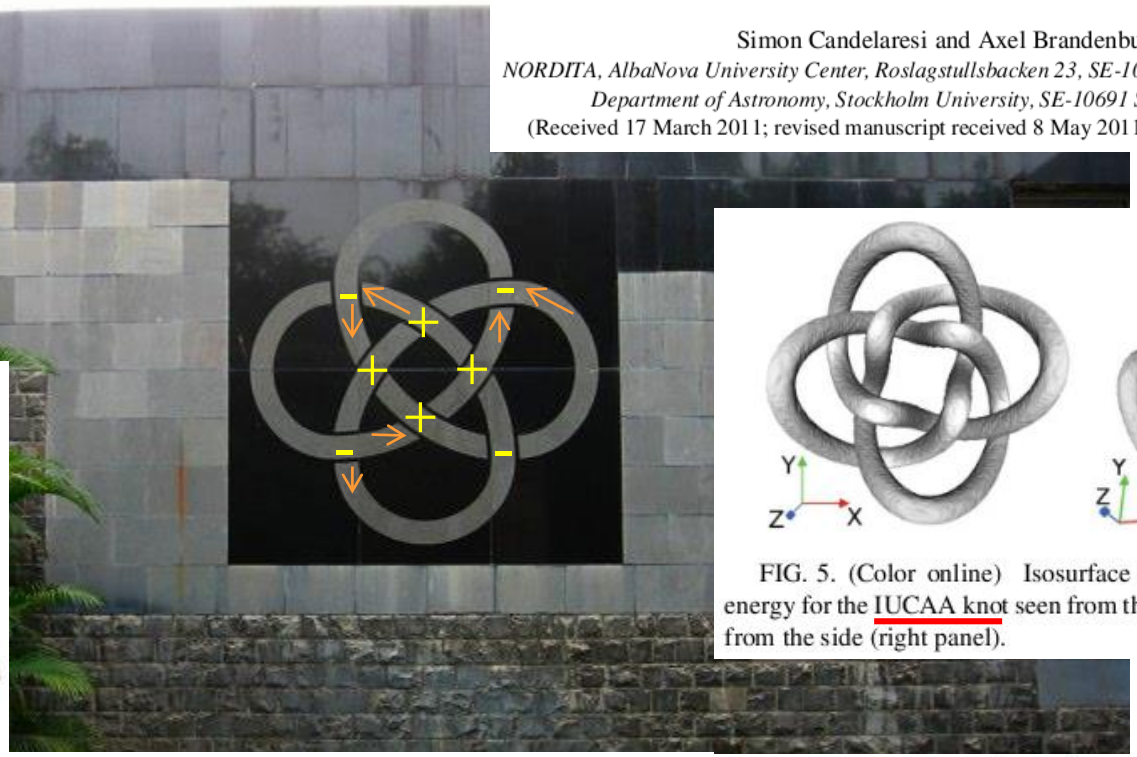
(Received 17 March 2011; revised manuscript received 8 May 2011; published 25 July 2011)







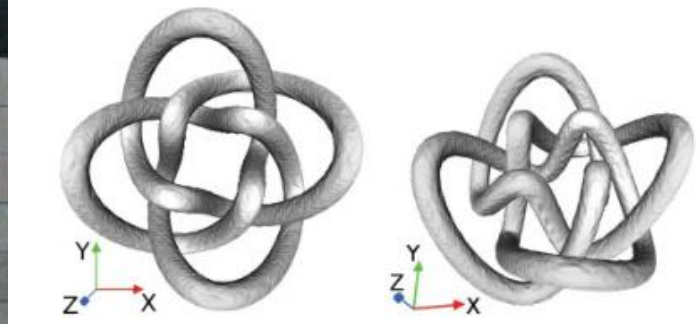
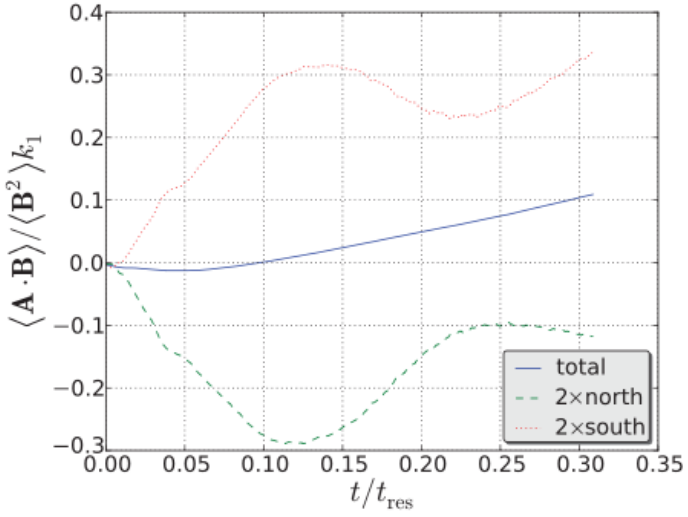
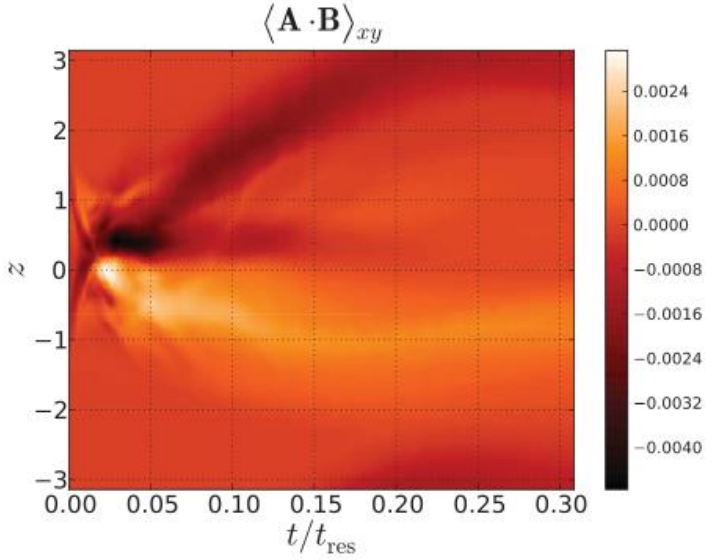


FIG. 5. (Color online) Isosurface of the initial magnetic field energy for the IUCAA knot seen from the top (left panel) and slightly from the side (right panel).





*In astrophysics: **self-excited** dynamos*

- Self-excited dynamos in industry
  - Soren Hjorth (1854), Sam Varley (1866), W. von Siemens, Ch. Wheatstone (indep: 1877)
- Astrophysics: Larmor (1919), but Cowling (1933), then Herzenberg (1958), Bacchus, ...
- Mean-field dynamos (Parker 1955, Steenbeck, Krause, Rädler (1966)
  - galaxies: Vainshtein & Ruzmaikin (1971)
- Small-scale dynamos (Kazantsev 1968)

Cowling's theorem (modern form)

« An **axisymmetric magnetic field** **cannot** be generated by dynamo action under the assumption of **axisymmetry** of
- velocity field,
- electrical conductivity,
- magnetic permeability and
- shape of the conductor »

(Kaiser & Tilgner, 2014)

THE MAGNETIC FIELD OF SUNSPOTS.

Sir Joseph Larmor, F.R.S. (1934)

In *M.N.* for 1933 November (pp. 39–48) there is a paper by Dr. T. G. Cowling on "The Magnetic Field of Sunspots." The Summary appended to it is brief and reads as follows :—

"The theory proposed by Sir Joseph Larmor, that the magnetic field of a sunspot is maintained by the currents it induces in moving matter, is examined and shown to be faulty : the same result also applies for the similar theory of the maintenance of the general field of Earth and Sun. The possibility that the sunspot field may arise as a disturbance in the general field is examined, and it is shown that several of the properties of the spot field are explicable on this hypothesis. Observation, however, must ultimately decide on its validity."

The view that I advanced briefly and tentatively long ago,* which has come to be referred to as, perhaps too precisely, the self-exciting dynamo analogy, is still, so far as I know, the only foundation on which a gaseous body such as the Sun could possess a magnetic field : so that if it is demolished there could be no explanation of the Sun's magnetism even remotely in sight. I am, however, not ready to accept the present adverse decision : and a restatement, not so condensed, perhaps with rather different emphasis, may be permitted.†

EFFECT OF INTERNAL MOTIONS ON THE DECAY OF A MAGNETIC FIELD IN A FLUID CONDUCTOR*

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Enrico Fermi Institute, University of Chicago
Received March 27, 1956

ABSTRACT

The decay of a magnetic field in a fluid conductor with internal motions is considered in case the magnetic and velocity fields have symmetry about an axis The underlying characteristic value problems are solved for certain simple velocity fields The method of solution is based on a new classification of the basic axisymmetric modes of the magnetic field in terms of Gegenbauer polynomials. The principal conclusion of the paper is that velocity fields of reasonable patterns and magnitudes can alter the time of decay that will obtain in the absence of motions by quite large factors. The bearing of this result on the problem of the origin of the earth's magnetic field is briefly discussed Certain other related questions are also considered.

of the order of 50 or more. To the question raised in § 2: "Can the time of decay of 14000 years, in the absence of internal motions, be prolonged to 500000 years (say) by velocities of reasonable magnitudes and patterns?" the answer would seem to be "Yes"; and if the answer is "Yes," the subsequent remarks in § 2 relative to the origin of the earth's magnetic field should have substance.

It was first suggested by Larmor (1919) as a possible explanation that the earth's magnetic field might be maintained by fluid motions in the core in the manner of a self-exciting dynamo. This suggestion received a serious setback when Cowling (1934; see also Backus and Chandrasekhar 1956) proved that steady dynamo action is impossible if the magnetic field and the fluid motions have axial symmetry. The possibility, nevertheless, of dynamo action has been explored, intensively, in recent years by Bullard and Gellman (1954), by Parker (1955), and by Elsasser (1955, 1956).

Parker's early work on the subject

THE FORMATION OF SUNSPOTS FROM THE SOLAR TOROIDAL FIELD*

EUGENE N. PARKER
Department of Physics, University of Utah, Salt Lake City, Utah
Received October 18, 1954

HYDROMAGNETIC DYNAMO MODELS

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Department of Physics, University of Utah, Salt Lake City, Utah
Received October 18, 1954; revised May 11, 1955

To obtain a quantitative estimate of the buoyancy force, consider a flux tube of 100 gauss at a depth of 2×10^4 km in the sun. At this level $\rho_e \cong 2.5 \times 10^{-4}$ gm/cm^3,

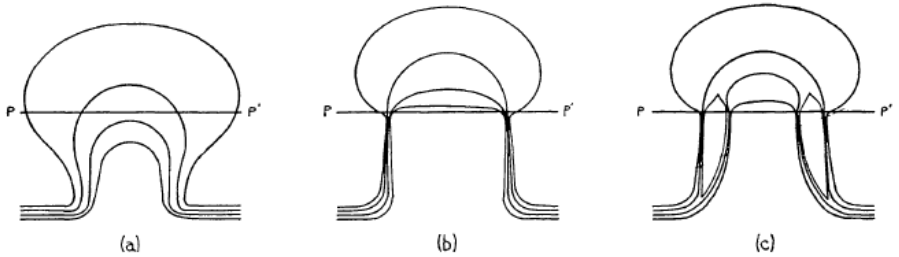


FIG. 2.—The development of a toroidal flux tube into a sunspot. *a*, indicates in a rough way how tube might look after being borne to the surface by the magnetic bouyancy; *b*, shows the concentra just under the photosphere due to cooling; *c*, indicates splitting of the tube as a consequence of the abı tapering above the cool region.

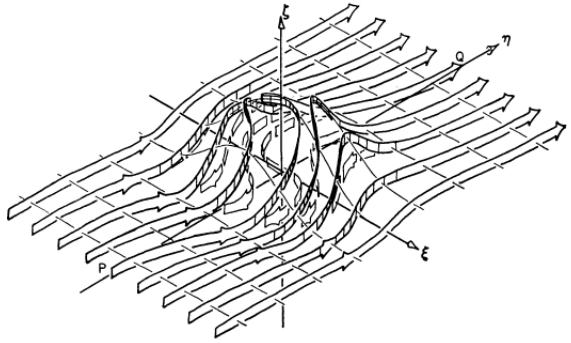


FIG. 1.—The deformation of a slab of toroidal field by cyclonic fluid motion. The ribbons underneath represent $-\beta$.

Equatorward migration

Solutions (113) of the dynamo equations become the migratory *dynamo waves*,

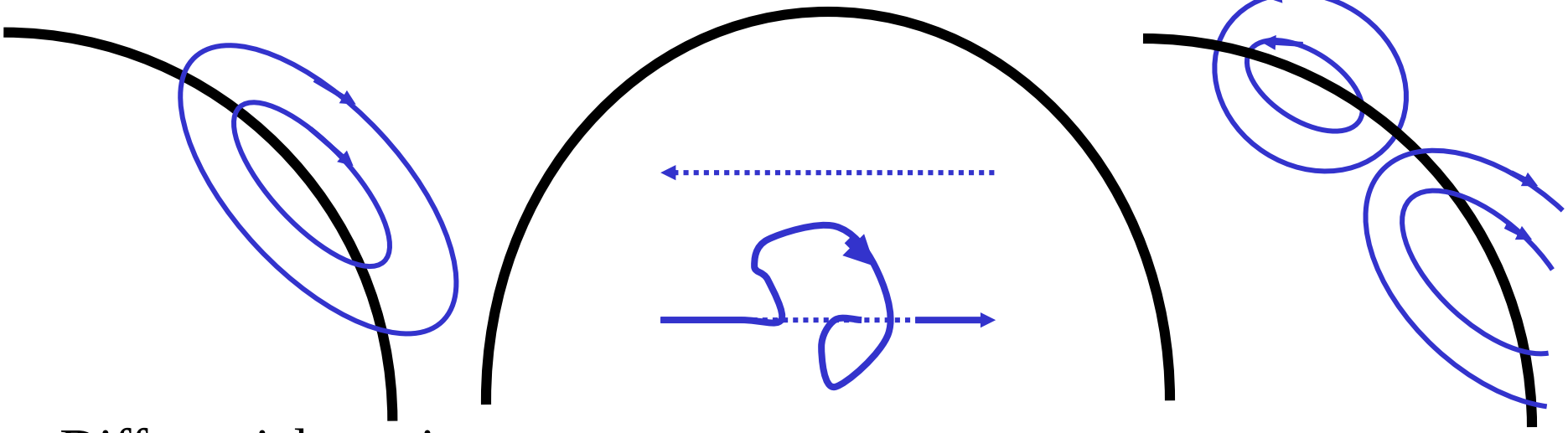
$$B = B_0 \exp\left[\left(\Omega - \frac{k^2}{\mu\sigma}\right)t\right] \exp\left[i\left(k\xi \pm \Omega t\right)\right],$$

$$A = -B_0 \frac{\Omega}{kH}\left(1 \mp i\right) \exp\left[\left(\Omega - \frac{k^2}{\mu\sigma}\right)t\right] \exp\left[i\left(k\xi \pm \Omega t\right)\right].$$

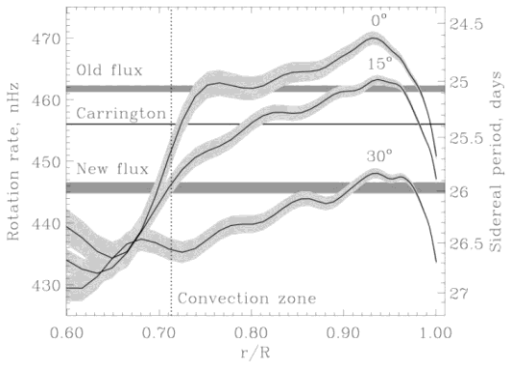
Crafoord prize in Astronomy 2020

"for pioneering and fundamental studies of the solar wind and magnetic fields from stellar to galactic scales"

αΩ dynamos (large scale)

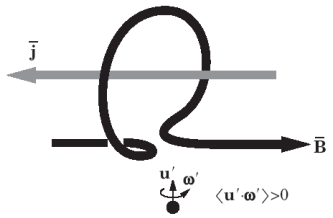


Differential rotation
(faster inside)

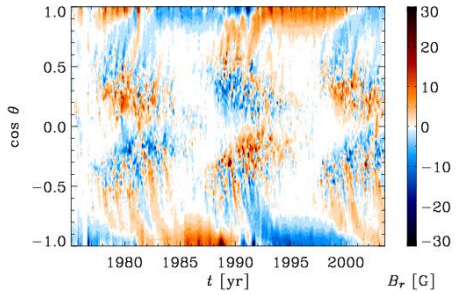


Cyclonic convection;
Buoyant flux tubes

→ α-effect

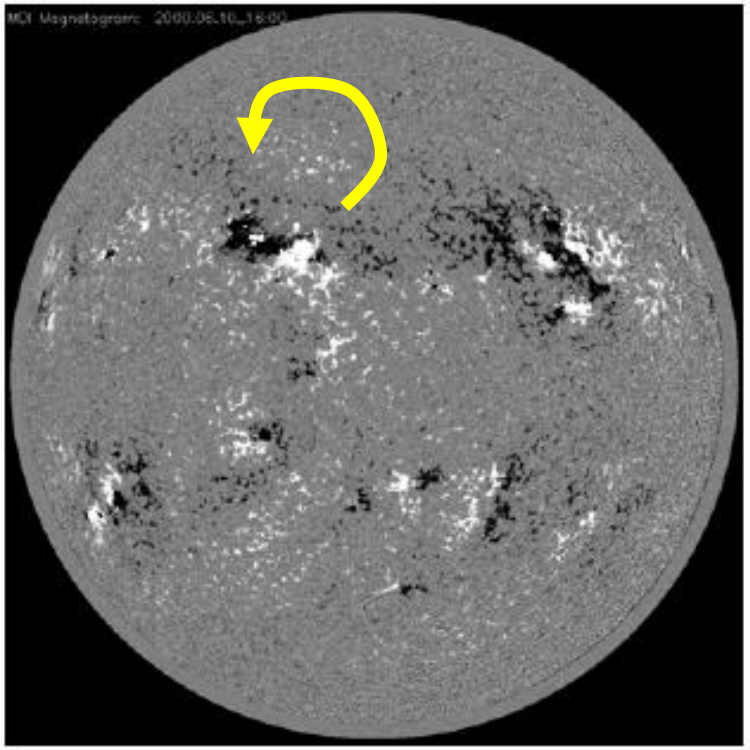


Equatorward
migration

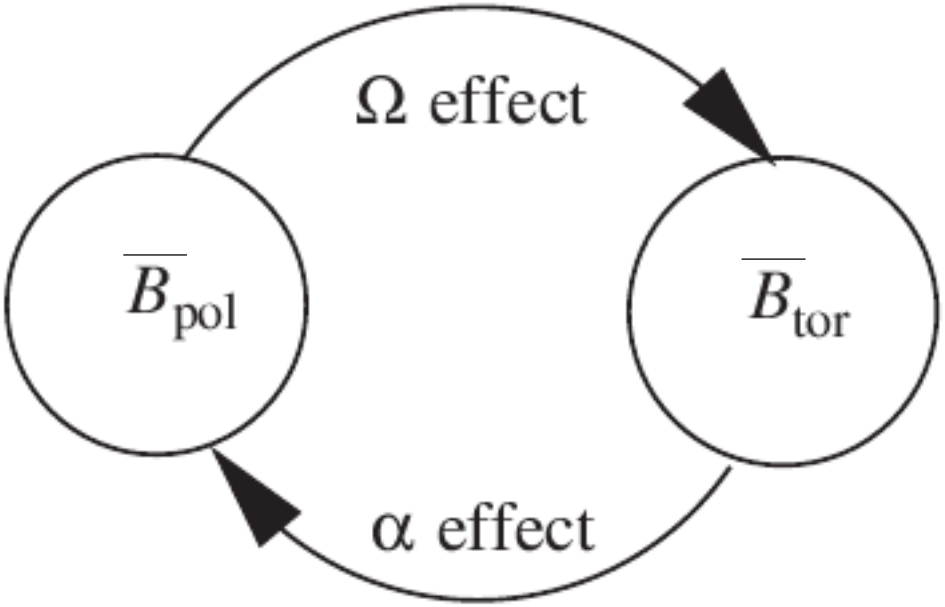


The Sun today and 9 years ago

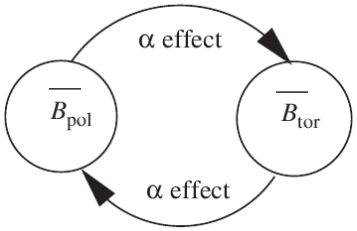
Solar magnetograms:
Line of sight B-field from circularly polarized light

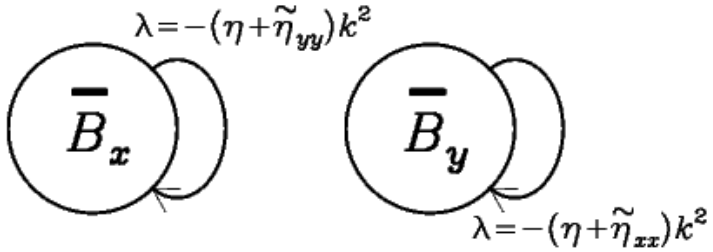


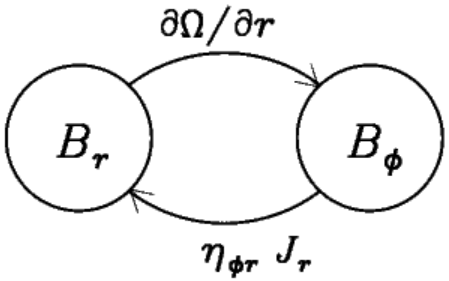
Cyclonic event: closing dynamo cycle



But not all dynamos of that type







Berechnung der mittleren Lorentz-Feldstärke $\overline{\mathfrak{v} \times \mathfrak{B}}$ für ein elektrisch leitendes Medium in turbulenter, durch Coriolis-Kräfte beeinflußter Bewegung

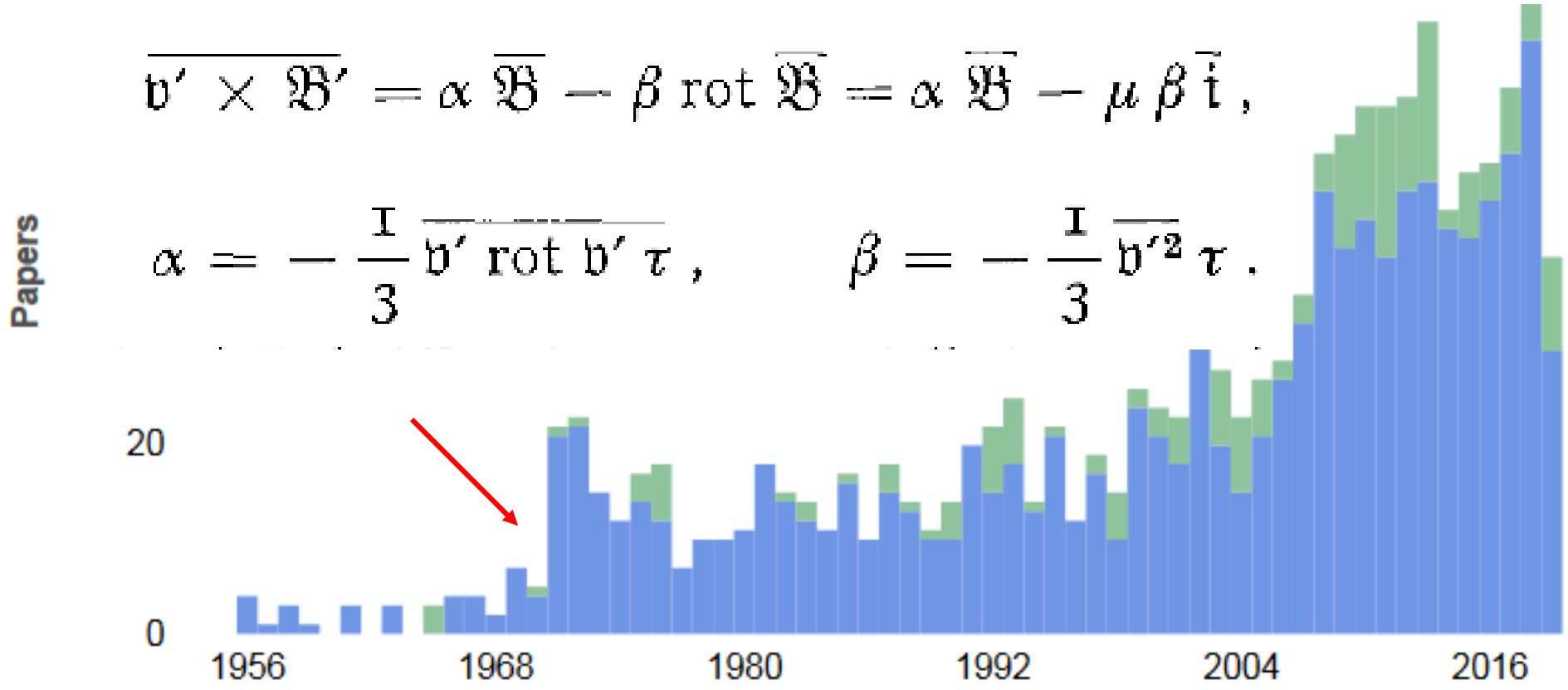
M. Steenbeck, F. Krause und K.-H. Rädler

Institut für Magnetohydrodynamik Jena der Deutschen Akademie der Wissenschaften zu Berlin

(Z. Naturforschg. **21 a**, 369—376 [1966]; eingegangen am 11. November 1965)

$$\overline{\mathfrak{v}' \times \mathfrak{B}'} = \alpha\, \overline{\mathfrak{B}} - \beta\, \mathrm{rot}\, \overline{\mathfrak{B}} = \alpha\, \overline{\mathfrak{B}} - \mu\, \beta\, \overline{\mathfrak{j}}\,,$$

$$\alpha = -\frac{1}{3}\, \overline{\mathfrak{v}'\, \mathrm{rot}\, \mathfrak{v}'}\, \tau\,, \qquad \beta = -\frac{1}{3}\, \overline{\mathfrak{v}'^2}\, \tau\,.$$



citations to Parker (1955)

The end of a generation has begun

1932-2020

Magnetic activity and variations in solar luminosity

E. A. Spiegel
Department of Astronomy, Columbia University, New York, New York 10027

N. O. Weiss*
Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138

1935-2020

chr. / AN **335**, No. 5, 459 – 469 (2014) / **DOI** 10.1002/asna.2

Mean-field dynamos: The old concept
Karl Schwarzschild Award Lecture 2013

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Leibniz-Institut für Astrophysik, An der Sternwarte 16, D-14482

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Published online 2014 Jun 2

Key words magnetic fields – magnetohydrodynamics (MHD)

1943-2020

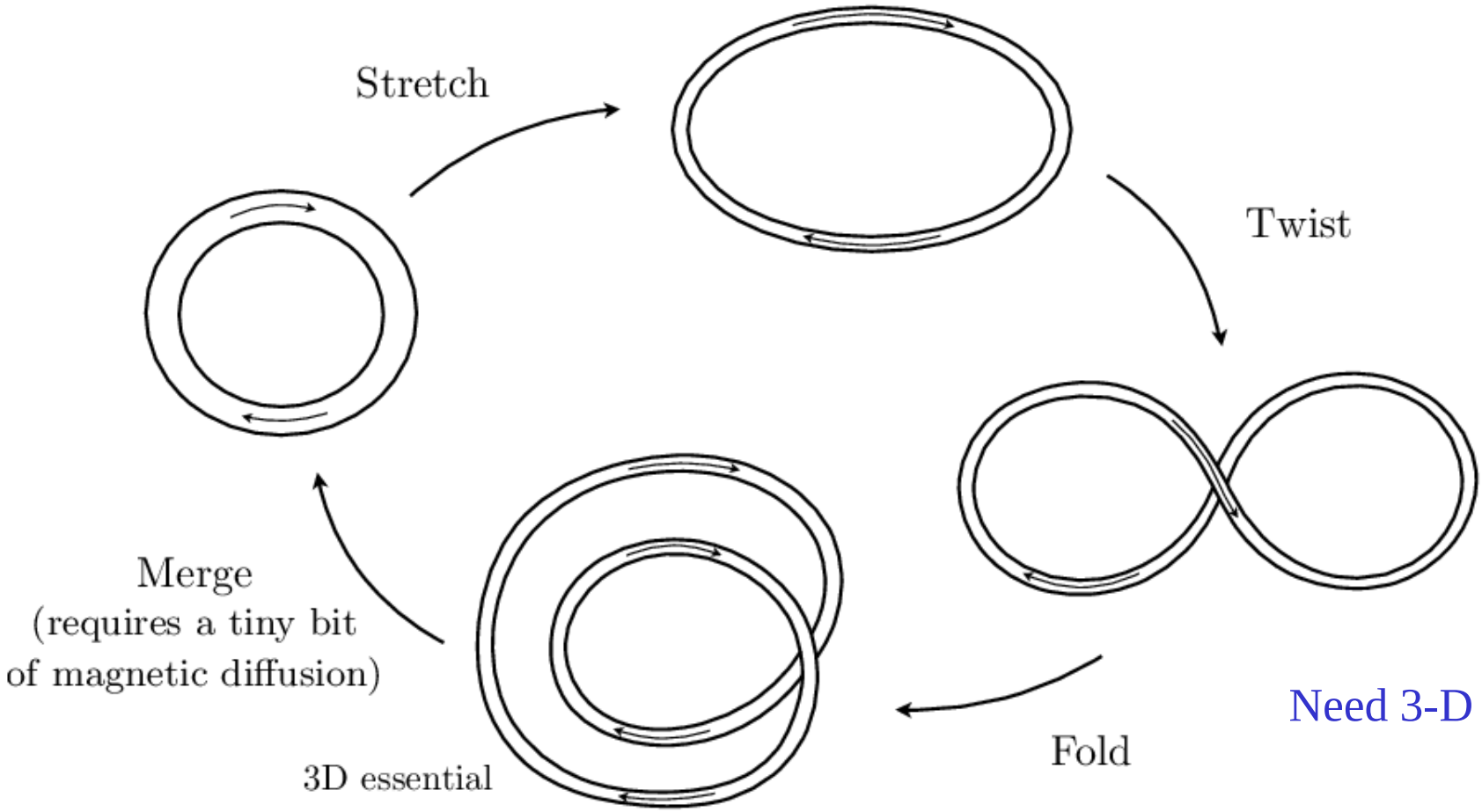
Mon. Not. R. Astron. Soc. **275**, 191–194 (1995)

On the generation of bisymmetric galaxies by tidal interactions

David Moss
Department of Mathematics, The University, Oxford Rd, Manchester M13

Accepted 1995 March 2. Received 1994 September 23

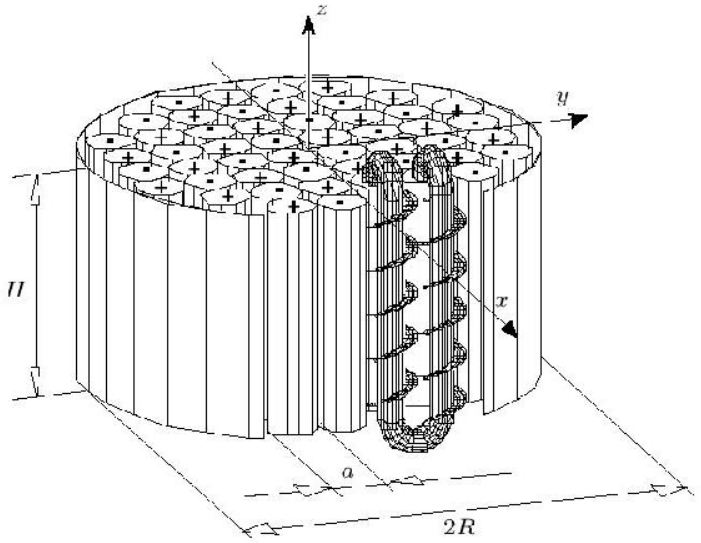
How can a dynamo work in principle?



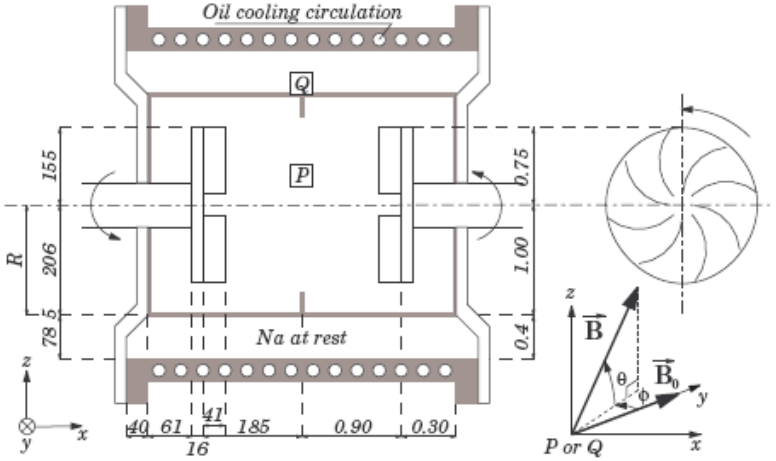
Need 3-D

Laboratory dynamo experiments

- Mean-field theory works for Karlsruhe Exp
- Cadarache: not explained by mean flow, but by mean electromotive force

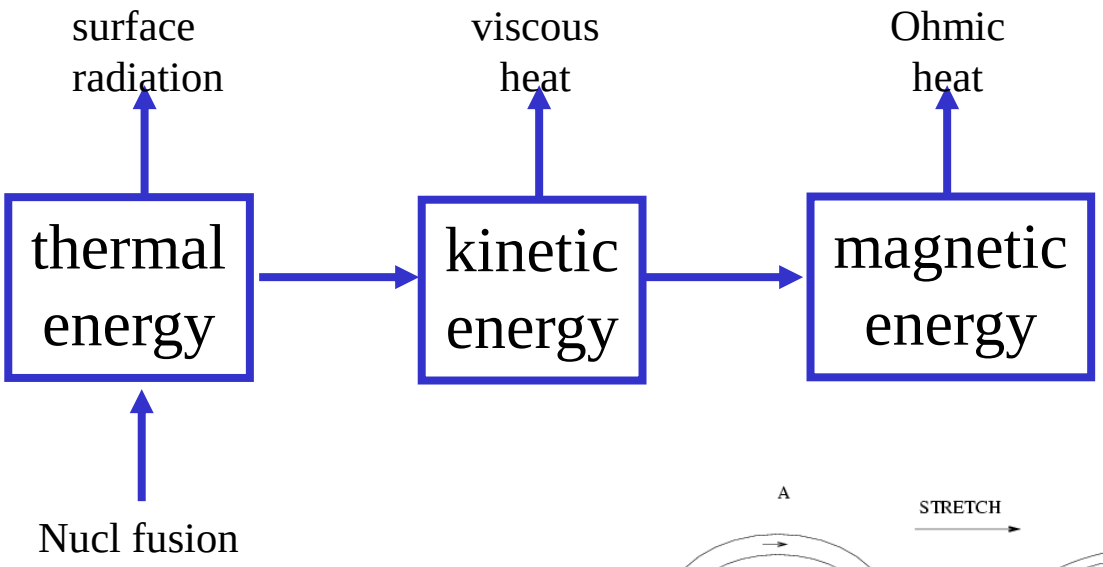


Karlsruhe (1999)

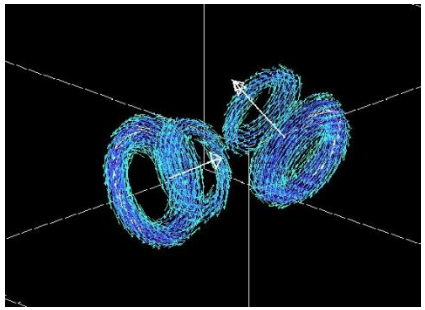


Cadarache (1999)

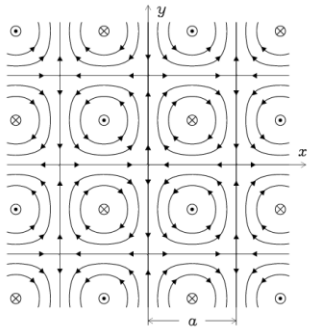
Dynamos: kinetic → magnetic energy



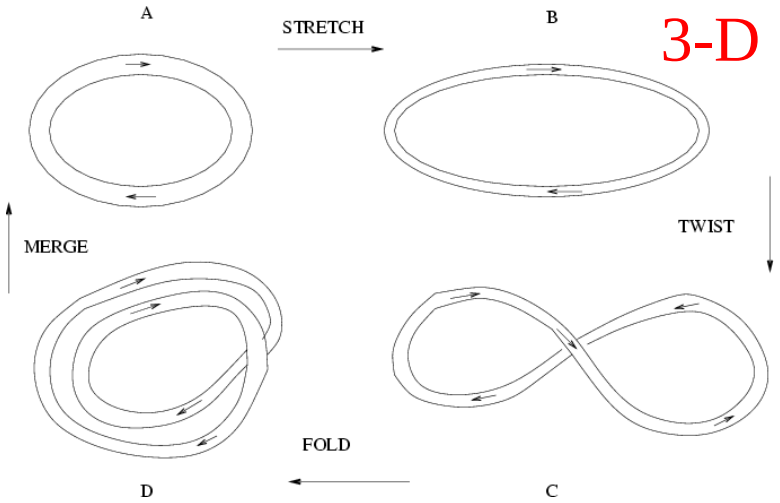
Need 3-D !!



Herzenberg (1999)



Karlsruhe (1999)



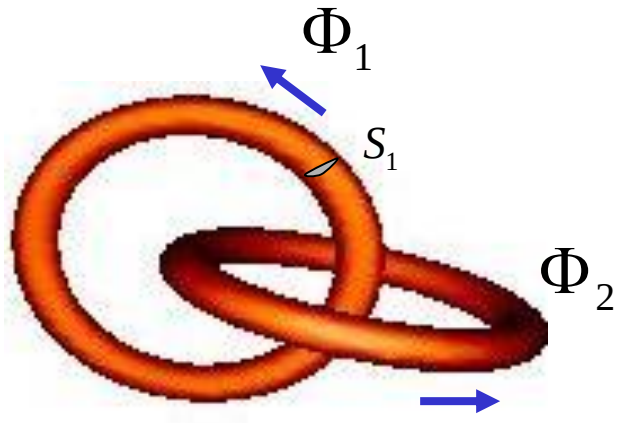
Stretch-Twist-Fold (1971)

MHD and magnetic helicity

$$\rho \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\nabla p + \mathbf{J}\times\mathbf{B} + \mathbf{F}, \qquad \frac{\partial \rho}{\partial t} = -\nabla\cdot\rho\mathbf{u}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{u}\times\mathbf{B} - \eta\mathbf{J}\right), \qquad \nabla\cdot\mathbf{B} = 0$$

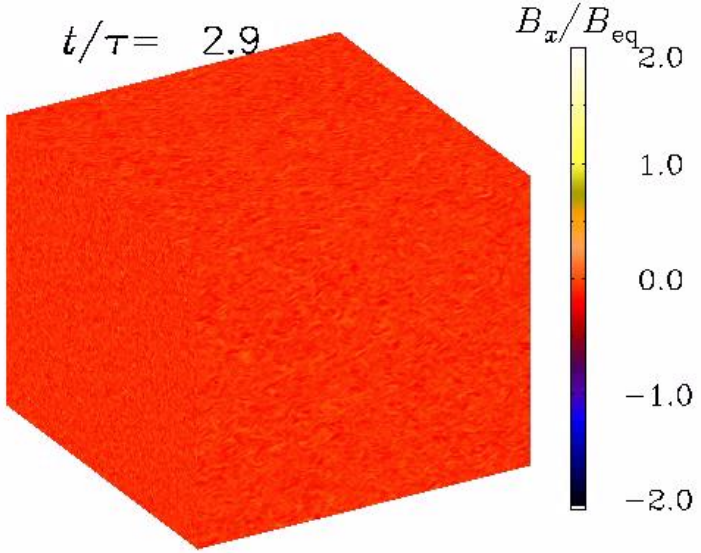
if kinematic

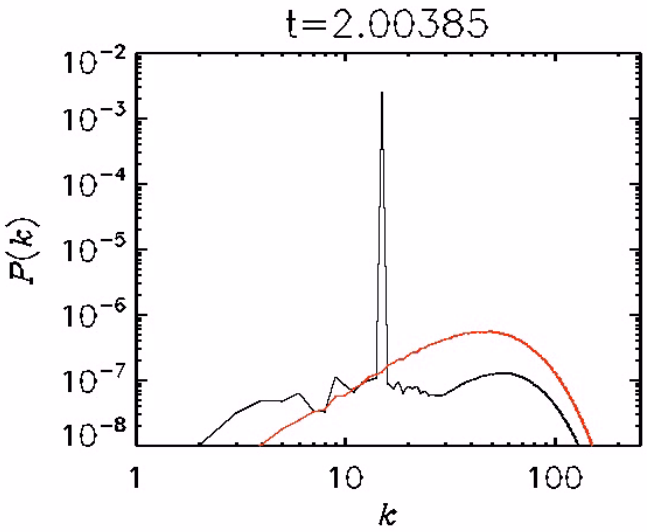


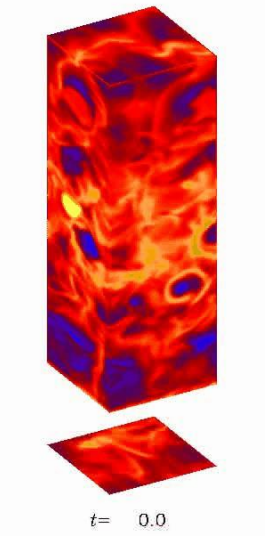
$$H = \int_V \mathbf{A}\cdot\mathbf{B}\,\mathrm{d}V$$

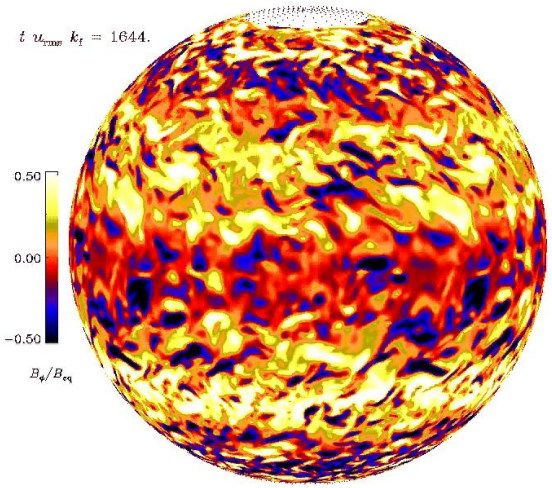
$$H = \pm 2\Phi_1\Phi_2$$

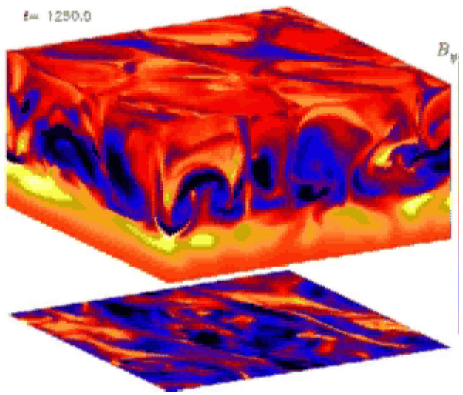
Modern numerical experiments

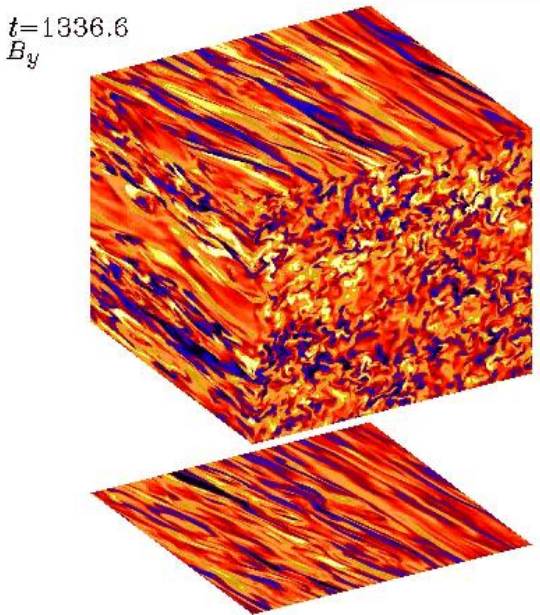




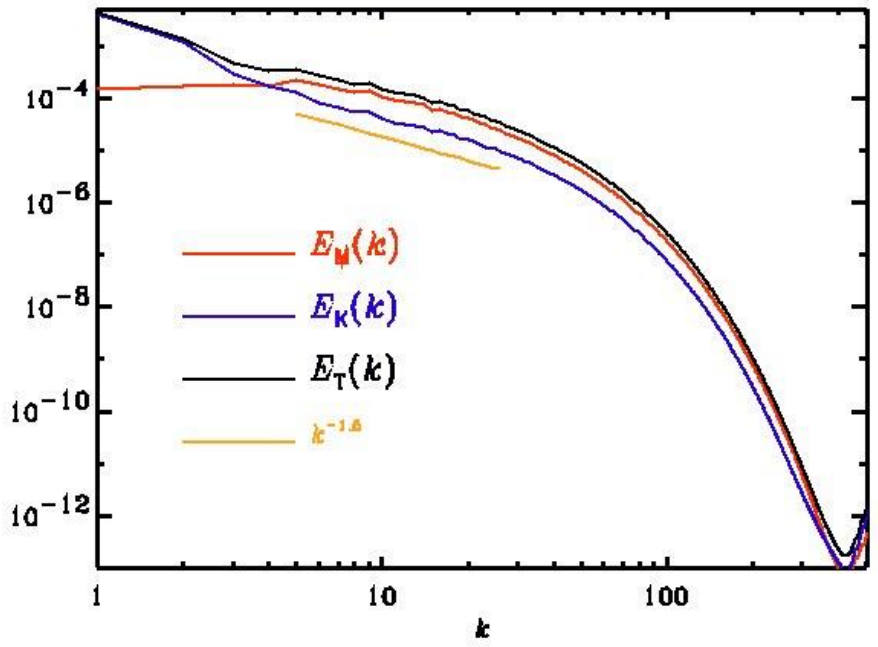


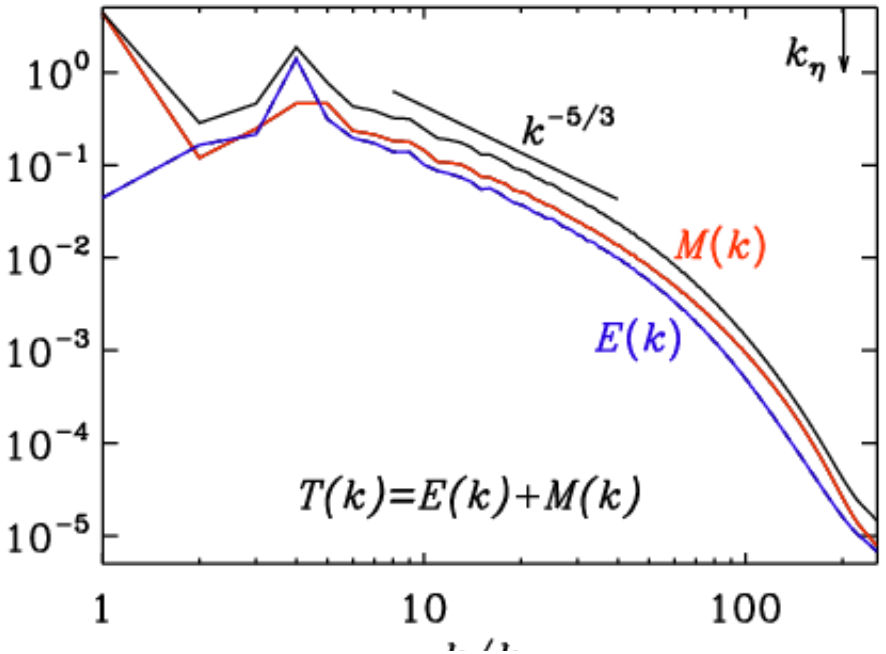


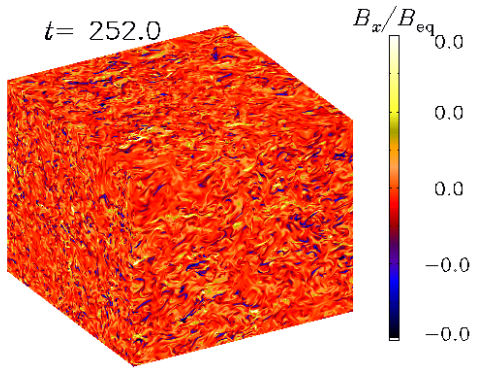


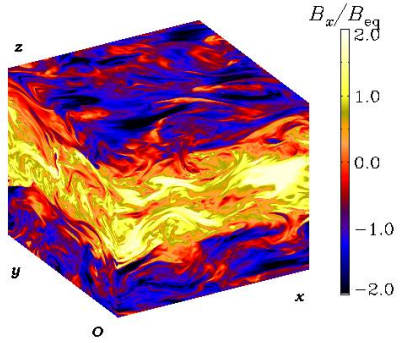


Small-scale vs large-scale dynamo



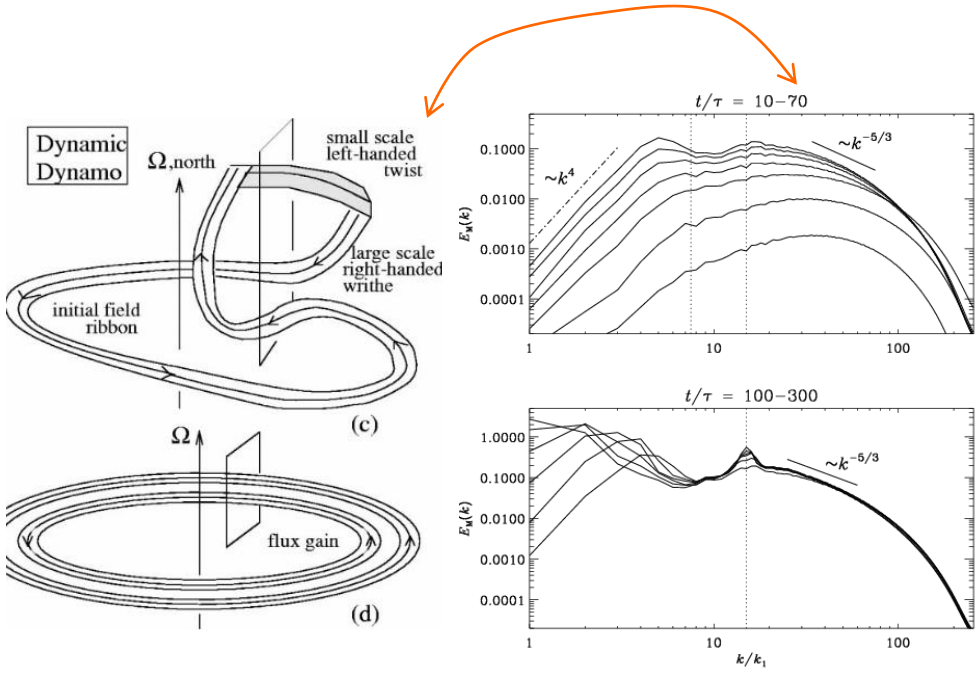


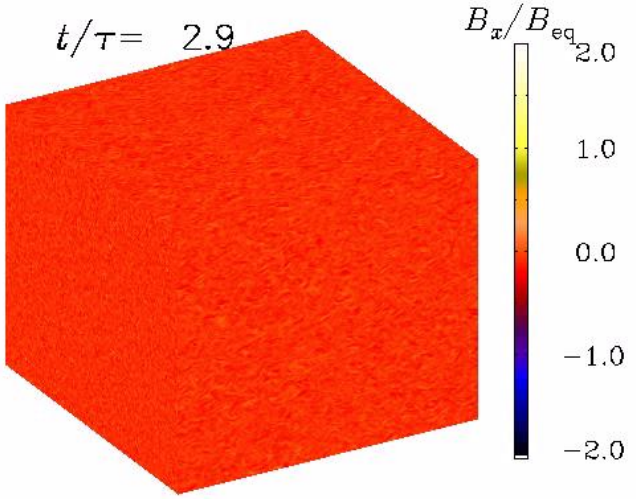




Turbulent dynamos unified

- Magnetic helicity conserved
- Inverse cascade produces small-scale waste!
- Opposite sign of helicity (or k)





Blackman & Brandenburg (2003)

Roberts (1972) flow(s)

$$u_x = v_0 \sin k_0 x \cos k_0 y, \qquad u_y = -v_0 \cos k_0 x \sin k_0 y,$$

while the components u_z are different and given by

$$u_z = w_0 \sin k_0 x \sin k_0 y \qquad \text{(flow I)}, \quad \rightarrow \text{helical}$$

$$u_z = w_0 \cos k_0 x \cos k_0 y \qquad \text{(flow II)}, \quad \rightarrow \text{nonhelical}$$

$$u_z = \tfrac{1}{2} w_0 (\cos 2k_0 x + \cos 2k_0 y) \qquad \text{(flow III)}, \quad \rightarrow \text{nonhelical}$$

$$u_z = w_0 \sin k_0 x \qquad \text{(flow IV)}, \quad \rightarrow \text{nonhelical}$$

resol	1/ν
8^3	5.14
16^3	5.51
32^3	5.52

Dynamo quiz: what dynamos are the last 3 flows?

Multiple choice: at least one of them is

A. also an α^2 dynamo

B. $\alpha\Omega$ dynamo

C. β effect dynamo (turbulent diffusivity)

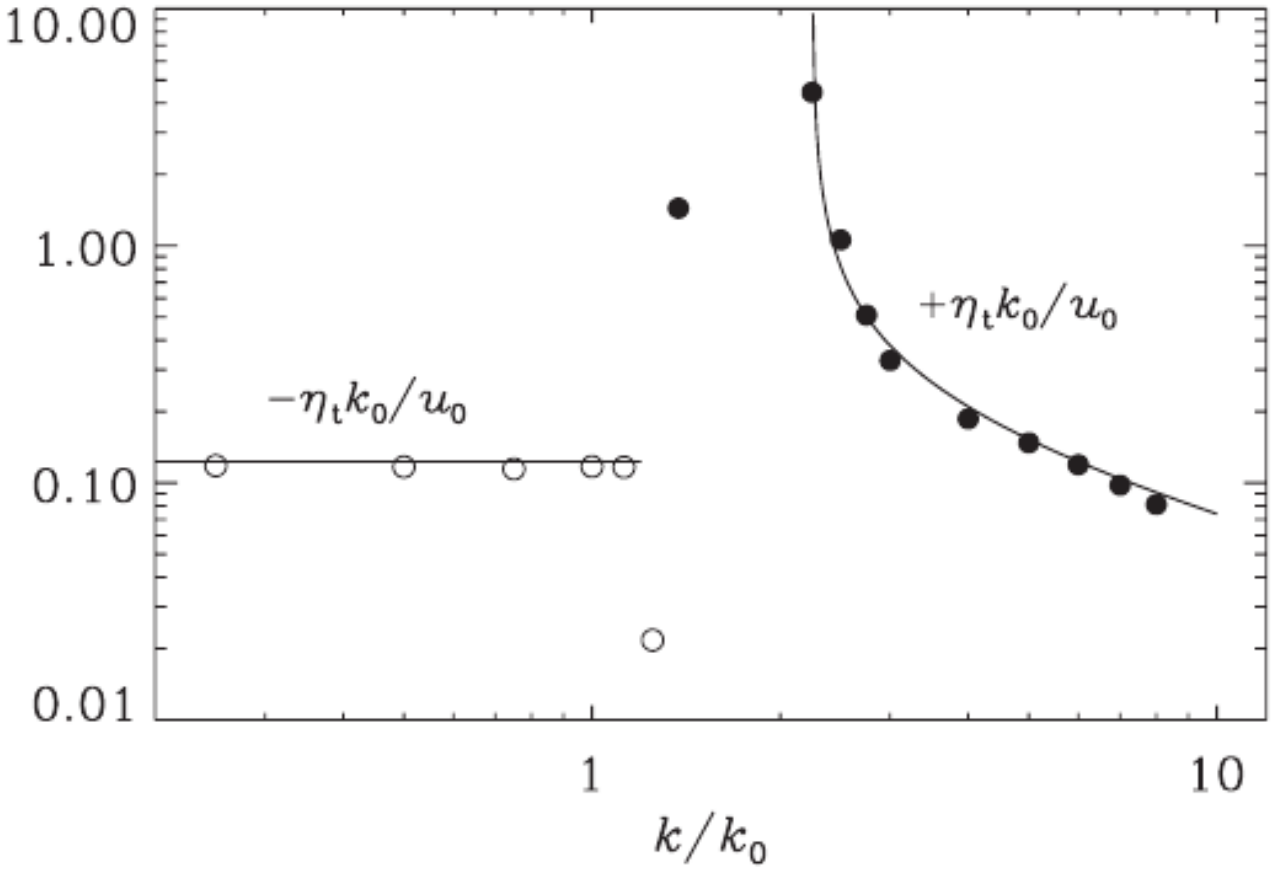
D. γ effect dynamo (pumping effect)

E. small-scale dynamo (ε effect)

Roberts flow IV: neg turb diff!

Yes, but: *k*-dependent; works only for small *k*

→ always positive diffusivity at large *k*



Multiple choice: at least one of them is

A. also an α^2 dynamo

B. $\alpha\Omega$ dynamo

C. β effect dynamo (turbulent diffusivity)

D. γ effect dynamo (pumping effect)

E. small-scale dynamo

How about the other Roberts flows

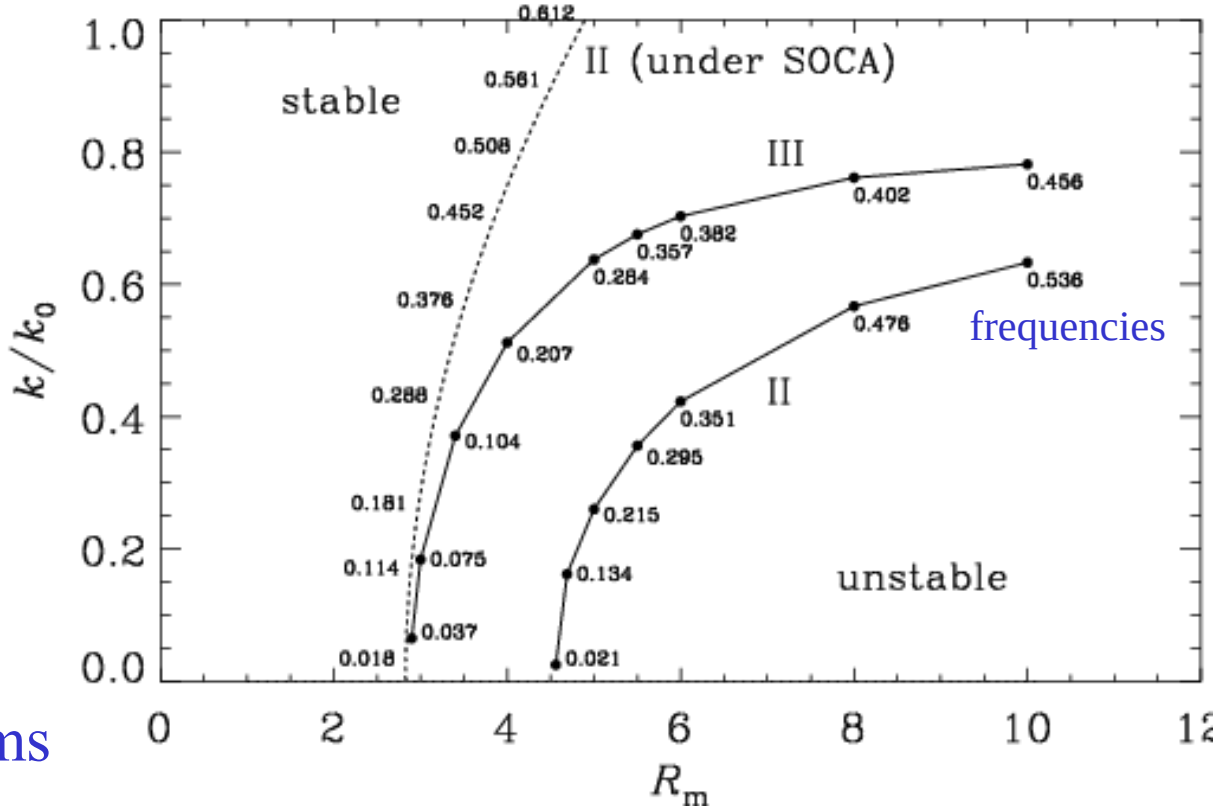
$$u_x = v_0 \sin k_0 x \cos k_0 y, \quad u_y = -v_0 \cos k_0 x \sin k_0 y,$$

while the components u_z are different and given by

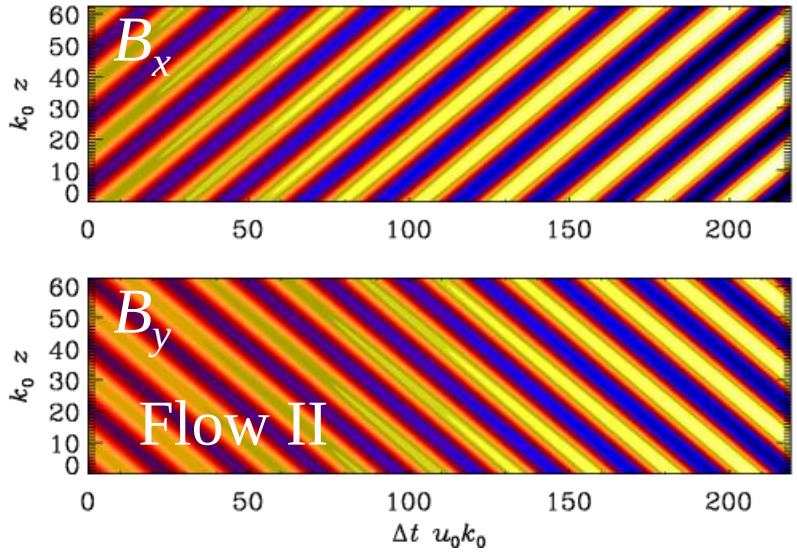
$u_z = w_0 \sin k_0 x \sin k_0 y$	(flow I),	→ helical
$u_z = w_0 \cos k_0 x \cos k_0 y$	(flow II),	→ nonhelical
$u_z = \frac{1}{2} w_0 (\cos 2k_0 x + \cos 2k_0 y)$	(flow III),	→ nonhelical
$u_z = w_0 \sin k_0 x$	(flow IV),	→ nonhelical

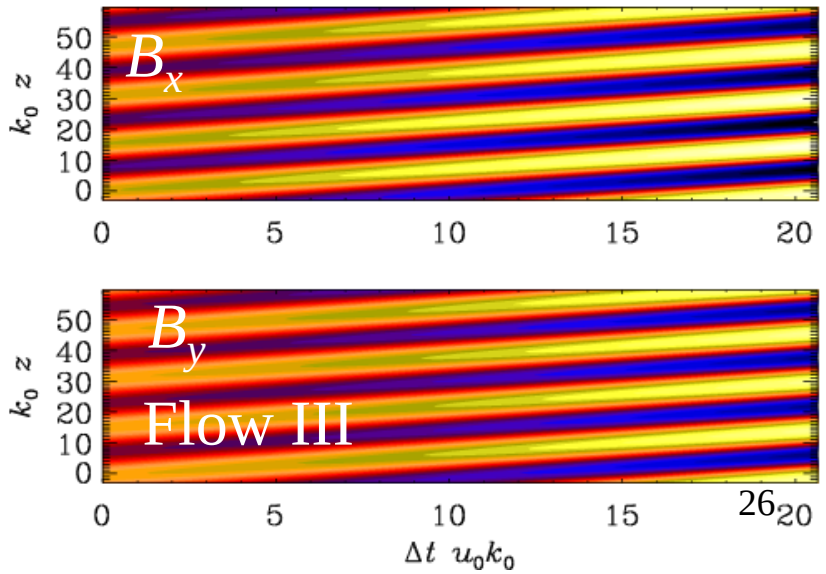
Rheinhardt, Devlen, Radler, & Brandenburg (2014)

Stability diagram & solutions



"Butterfly" diagrams





Understand with memory effect

$$p \approx -\frac{ik\gamma_0}{1-i\omega\tau} - (\eta + \tilde{\eta}_t)k^2$$

Small frequencies

$$\mathrm{Re}\, p + i\,\mathrm{Im}\, p = -ik\gamma_0(1+i\omega\tau) - (\eta + \tilde{\eta}_t)k^2$$

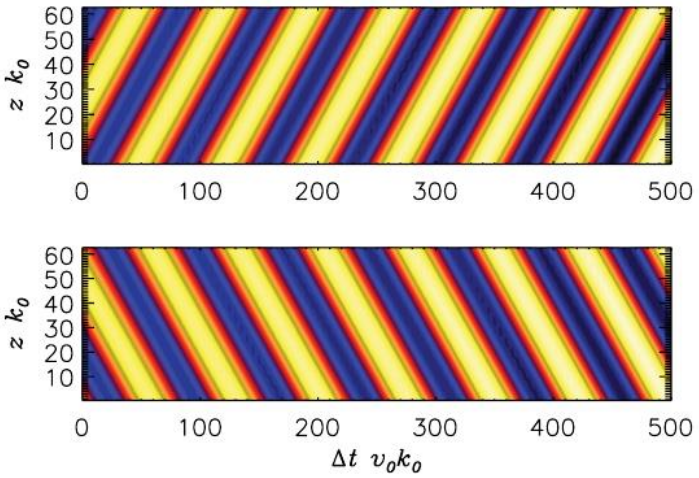
Real part

$$\mathrm{Re}\, p = k\gamma_0\omega\tau - (\eta + \tilde{\eta}_t)k^2$$

Imaginary part

$$\mathrm{Im}\, p = -ik\gamma_0$$

$g.B \rightarrow u.B$



Multiple choice: at least one of them is

A. also an α^2 dynamo

B. $\alpha\Omega$ dynamo

C. β effect dynamo (turbulent diffusivity)

D. γ effect dynamo (pumping effect)

E. small-scale dynamo

Axisymmetric dynamo action is possible with anisotropic conductivity

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[1] *Université Grenoble Alpes, Université Savoie Mont Blanc, CNRS, IRD, IFSTTAR, ISTerre, 38000 Grenoble, France*

[2] *Univ. Lyon, UCBL, ENS de Lyon, CNRS, UMR 5276 LGL-TPE, F-69622 Lyon, France*

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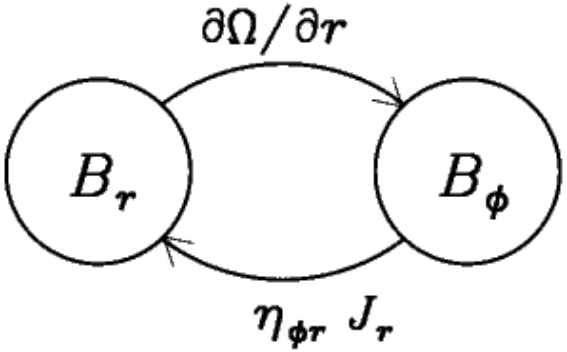
A milestone of dynamo theory is Cowling's theorem, known in its modern form as the impossibility for an axisymmetric velocity field to generate an axisymmetric magnetic field by dynamo action. Using anisotropic electrical conductivity, we show that an axisymmetric dynamo is in fact possible with a motion as simple as solid-body rotation. On top of that, the instability analysis can be conducted entirely analytically, leading to an explicit expression of the dynamo threshold, which is the only example in dynamo theory.



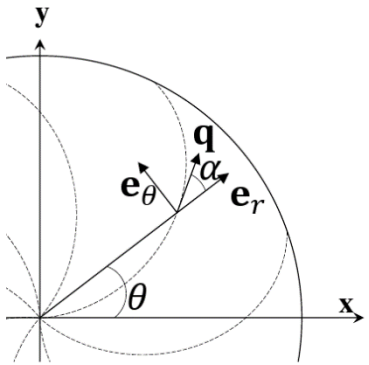
$$dA_\phi/dt = -\eta_{\phi r} J_r$$

We choose a logarithmic spiral anisotropy

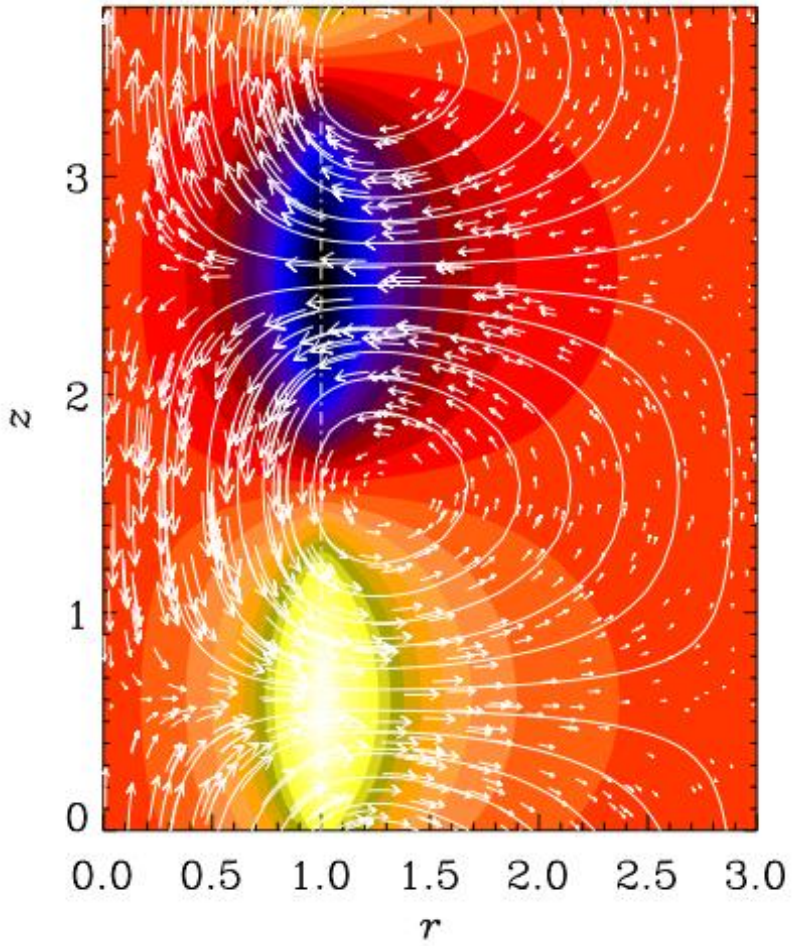
$$\mathbf{q} = \cos\alpha\, \mathbf{e_r} + \sin\alpha\, \mathbf{e_\theta}$$

$$[\sigma] = \begin{pmatrix} \sigma^{\parallel}c^2 + \sigma^{\perp}s^2 & (\sigma^{\parallel} - \sigma^{\perp})cs & 0 \\ (\sigma^{\parallel} - \sigma^{\perp})cs & \sigma^{\parallel}s^2 + \sigma^{\perp}c^2 & 0 \\ 0 & 0 & \sigma^{\perp} \end{pmatrix}_{(\mathbf{e_r}, \mathbf{e_\theta}, \mathbf{e_z})}$$

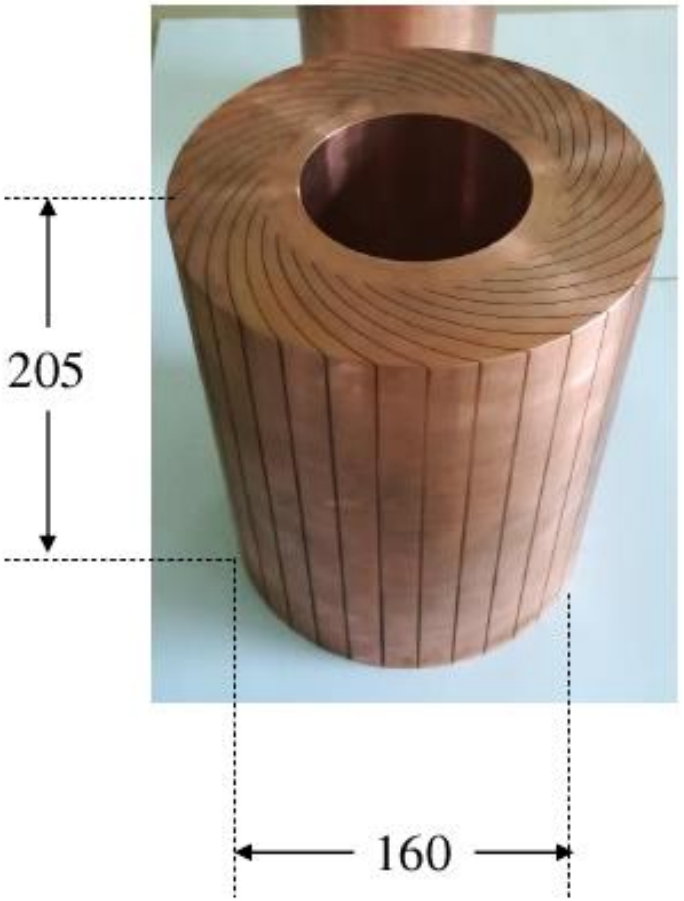
$$(c, s) = (\cos\alpha\,, \sin\alpha)$$



Axisymmetric dynamos: B=B(r,z)

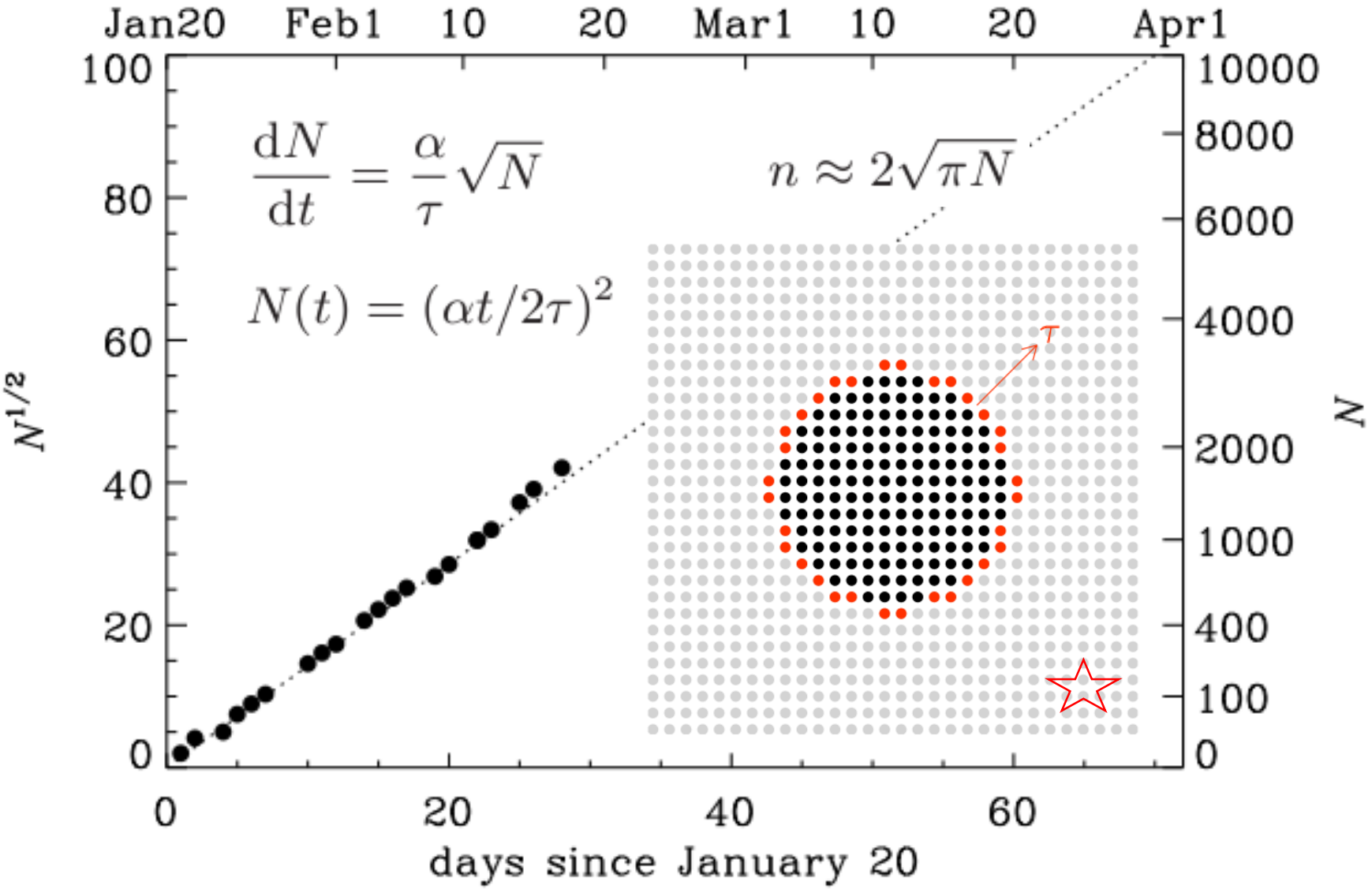


Pencil Code solution
(with Matthias Rheinhardt)



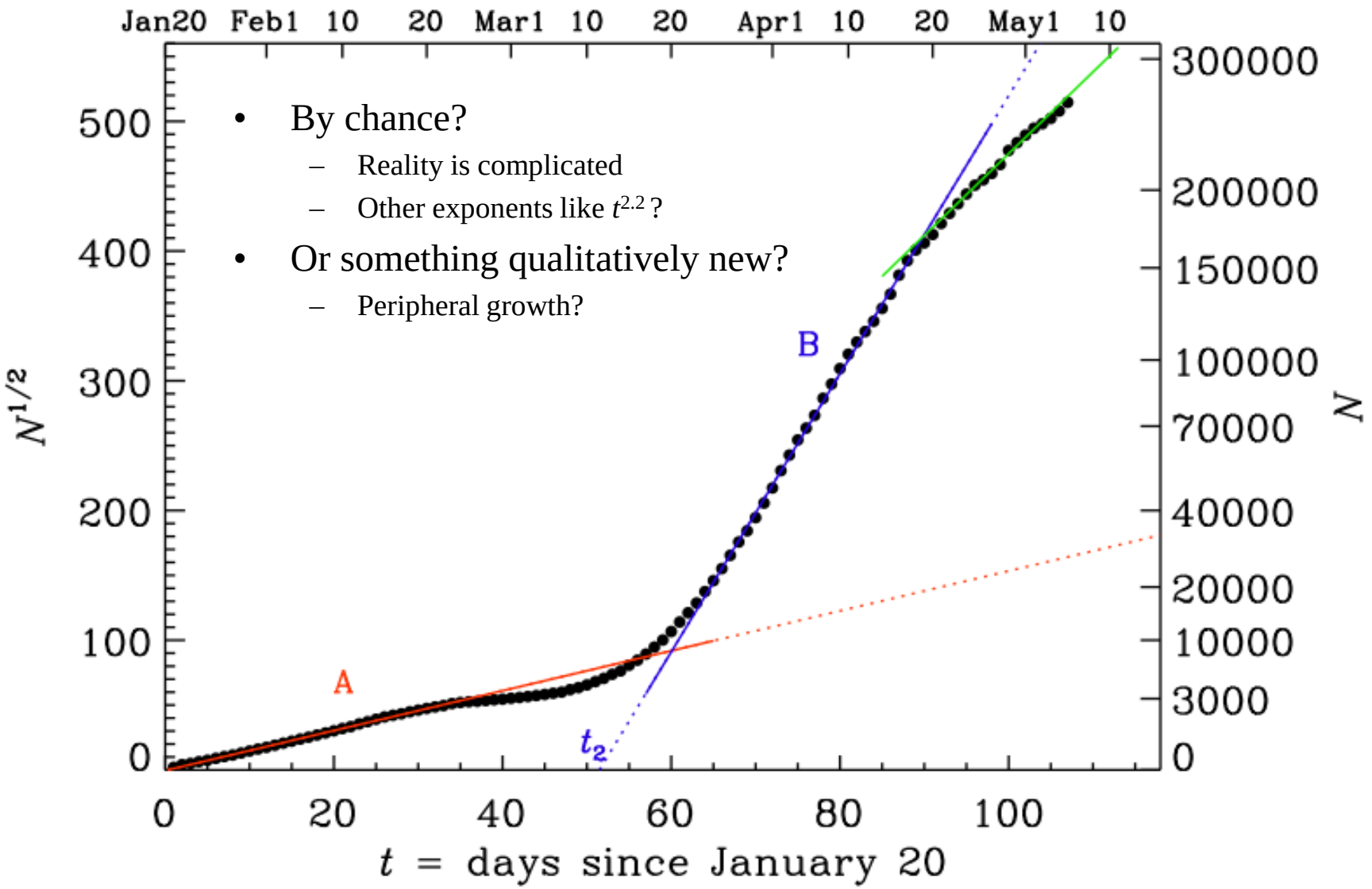
Experimental setup
(Alboussiere et al)

My take on Corona: peripheral growth



arXiv:2002.03638v2 [q-bio.PE] 14 Feb 2020

Piecewise quadratic growth



- By chance?
  - Reality is complicated
  - Other exponents like $t^{2.2}$?
- Or something qualitatively new?
  - Peripheral growth?

SIR model with spatial extent

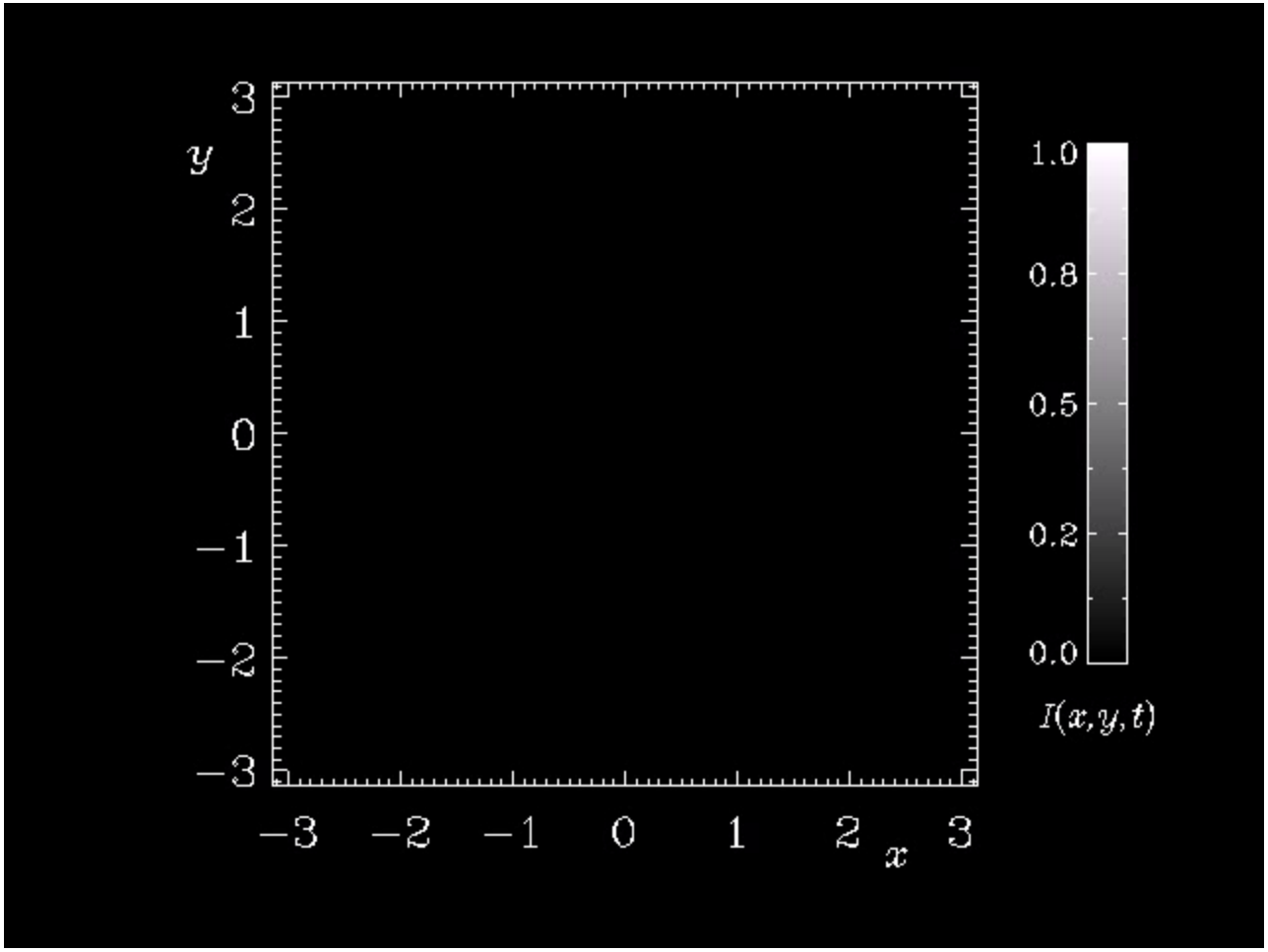
$$\frac{\partial S}{\partial t} = -\lambda S I,$$

$$\frac{\partial I}{\partial t} = \lambda S I - \mu I + \kappa \nabla^2 I,$$

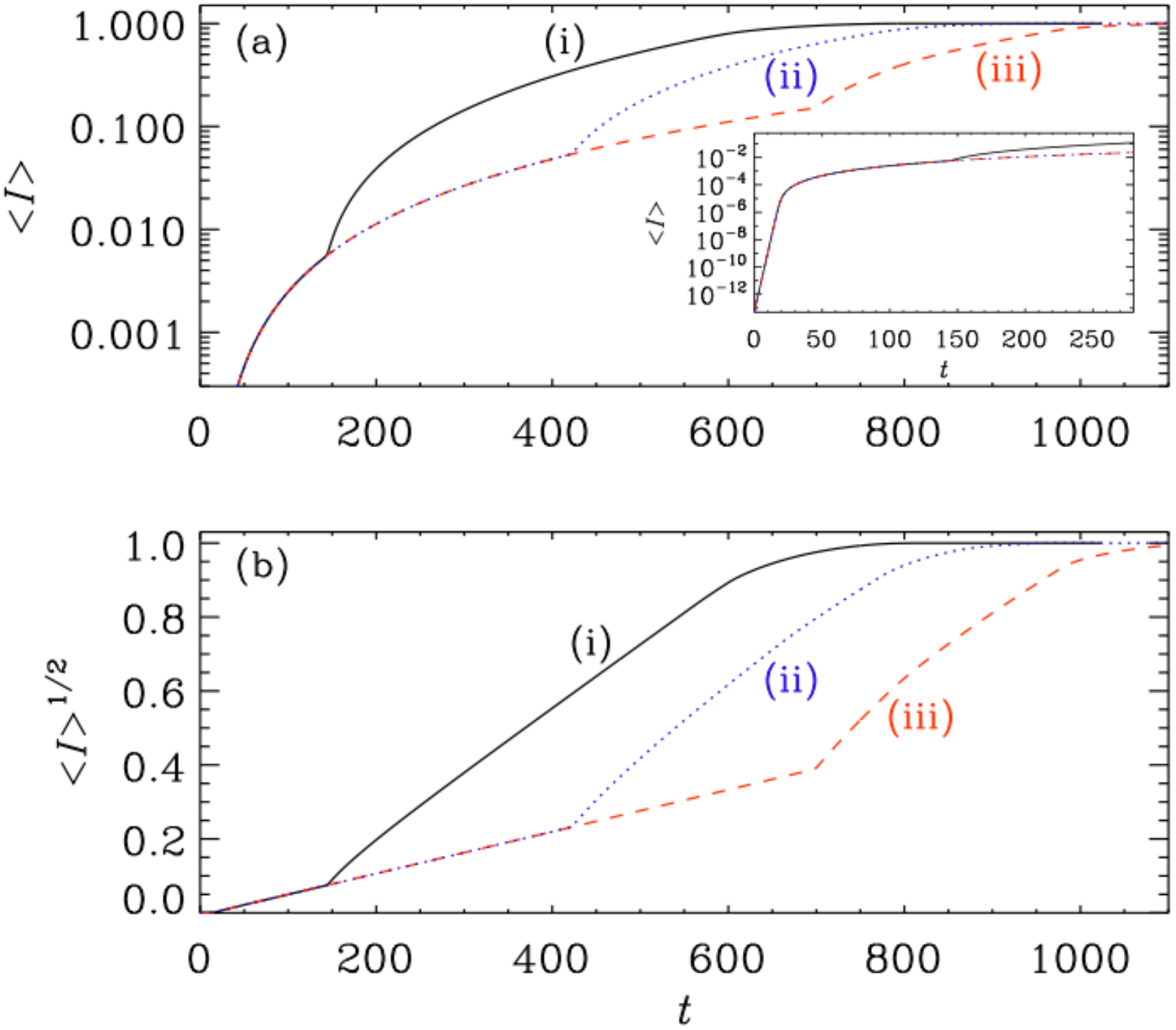
$$\frac{\partial R}{\partial t} = \mu I.$$

- Usually just exponential growth + saturation
  - Now also spreading (depends on λ and κ)
  - Constant front speed $c = 2\sqrt{\lambda\kappa}$ (Murray, 2003)
  - width $\sqrt{\kappa/\lambda} = 10^{-3}$.

SIR model with spatial extent



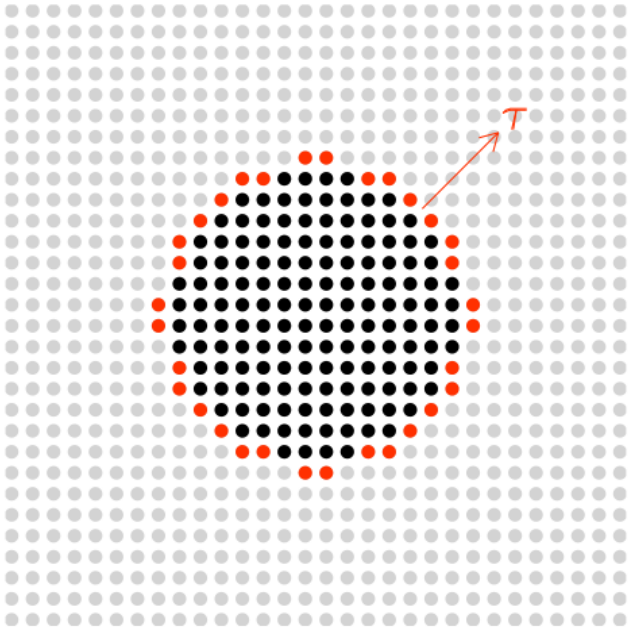
- Growth of one versus several hot spots



Piecewise quadratic growth perfectly confirmed

Conclusions

- Piecewise quadratic growth observed
  - Explained by peripheral growth
  - Second phase explained several hotspots
- Implications
  - Saturation locally
  - Subsequent spreading
- Can that be possible?
  - Everybody immune?
  - Or everybody isolated?
  - Current confinement



arxiv.org https://arxiv.org/abs/2002.03638