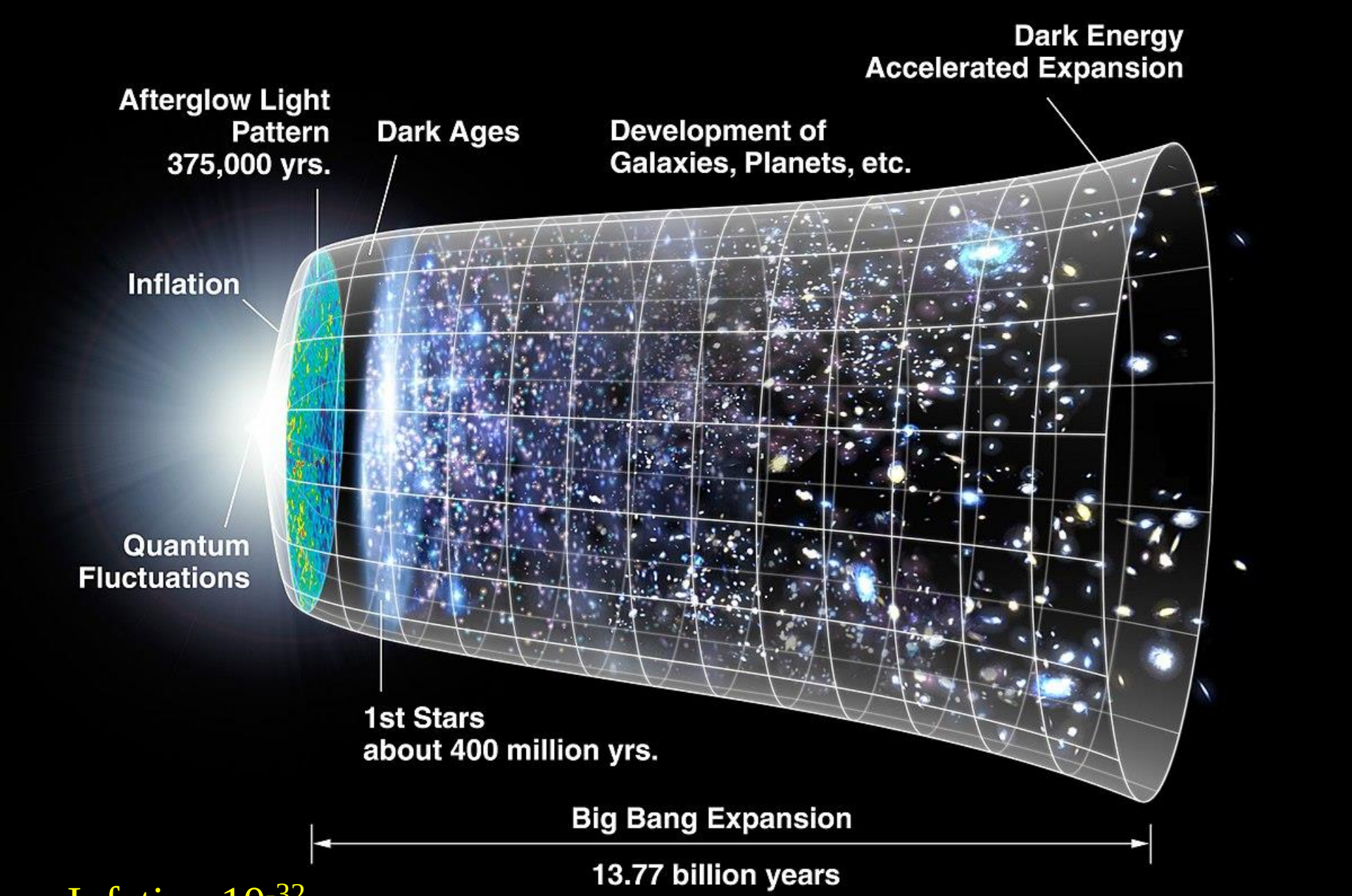
Polarized gravitational waves from early universe hydromagnetic turbulence

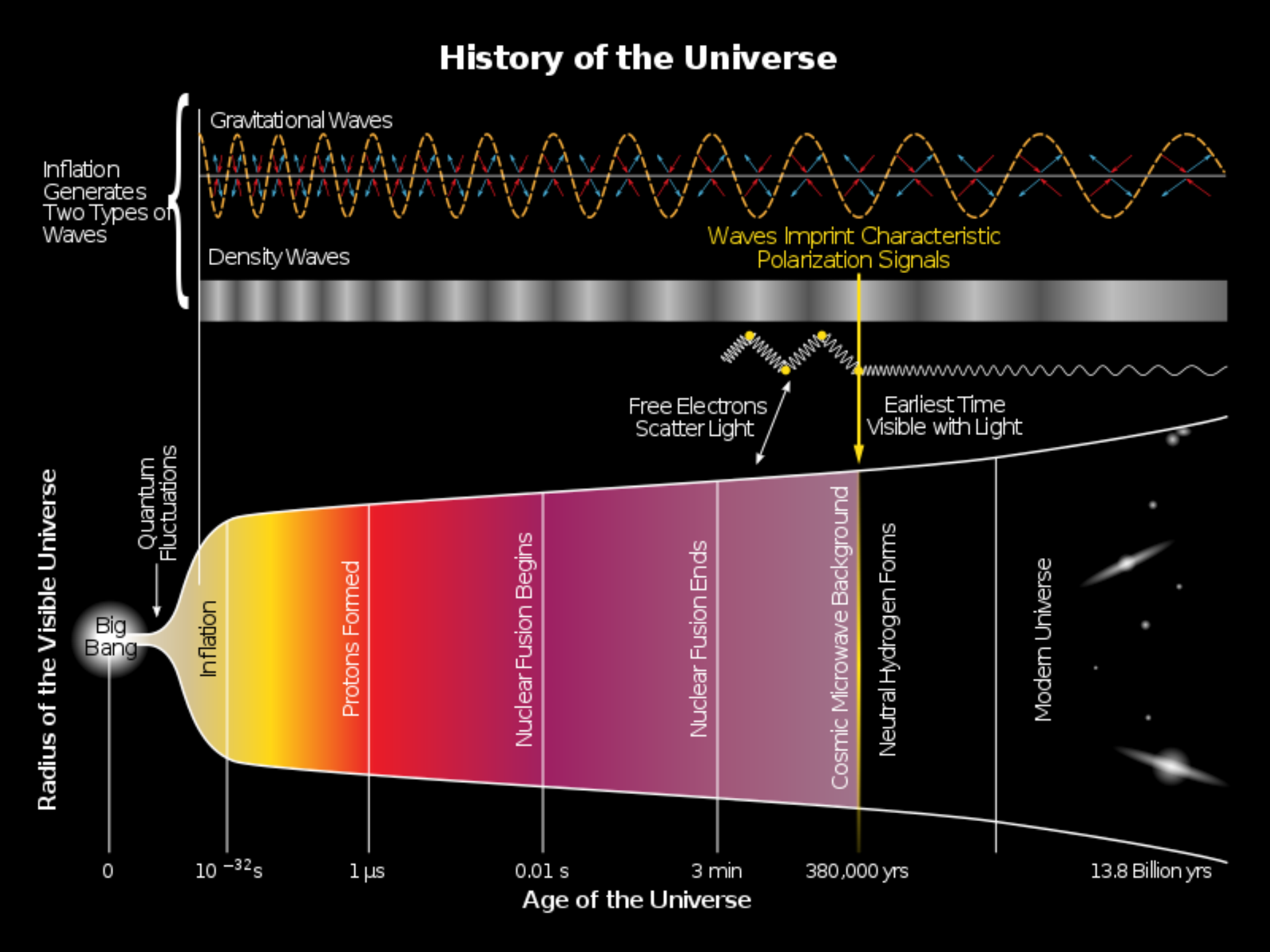
- Early universe turbulence
- Primordial magnetic fields
- Maxwell & Reynolds stresses drive gravitational waves
- Observable with LISA
- Spectra & polarization!

Axel Brandenburg, Tina Kahniashvili, Arthur Kosowsky, Sayan Mandal, & Alberto Roper Pol



Infation 10^{-32}s

Electroweak phase transition 10^{-11}s (10^{15}K)


History of the Universe
Gravitational Waves
Inflation Generates Two Types of Waves
Density Waves
Waves Imprint Characteristic Polarization Signals
Free Electrons Scatter Light
Earliest Time Visible with Light
Quantum Fluctuations
Big Bang
Inflation
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Neutral Hydrogen Forms
Modern Universe
Radius of the Visible Universe
0
10 −32s
1 µs
0.01 s
3 min
380,000 yrs
13.8 Billion yrs
Age of the Universe

History of gravitational waves

- Heaviside (1893): analogy with electromagnetism
- Poincare (1905): emanating from body at speed of light
- Einstein (1916, 1918), three types of waves
- Eddington (1922): two of three are artifacts
- Einstein & Rosen (1936) unphysical altogether
- referee Robertson: harmless coordinate singularities
- Pirani (1956): manifestly gauge-invariant observables
- Hulse & Taylor (1975): → indirect GW detection
- GW150914. 2017 Nobel prize to Weiss, Thorne, Barish

Confused situation by 1936

July 27, 1936

Dear Sir.

"We (Mr. Rosen and I) had sent you our manuscript for publication and had not authorized you to show it to specialists before it is printed. I see no reason to address the—in any case erroneous—comments of your anonymous expert. On the basis of this incident I prefer to publish the paper elsewhere."

Respectfully

Einstein

An anecdote illustrating the confused situation prevailing at that time is given in Infeld's autobiography. Infeld refers to the day before a scheduled talk that Einstein was to give at Princeton on the "Nonexistence of gravitational waves." Einstein was already aware of the error in his manuscript, which was previously pointed out by Infeld. There was no time to cancel the talk. The next day Einstein gave his talk and concluded, *"If you ask me whether there are gravitational waves or not, I must answer that I don't know. But it is a highly interesting problem"* [10].

"Note—The second part of this article was considerably altered by me after the departure to Russia of Mr. Rosen as we had misinterpreted the results of our formula. I want to thank my colleague Professor Robertson for their friendly help in clarifying the original error. I also thank Mr. Hoffmann your kind assistance in translation."

In the end, Einstein became convinced of the existence of gravitational waves, whereas Nathan Rosen always thought that they were just a formal mathematical construct with no real physical meaning.

EVIDENCE FOR DISCOVERY OF GRAVITATIONAL RADIATION*

J. Weber
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 29 April 1969)

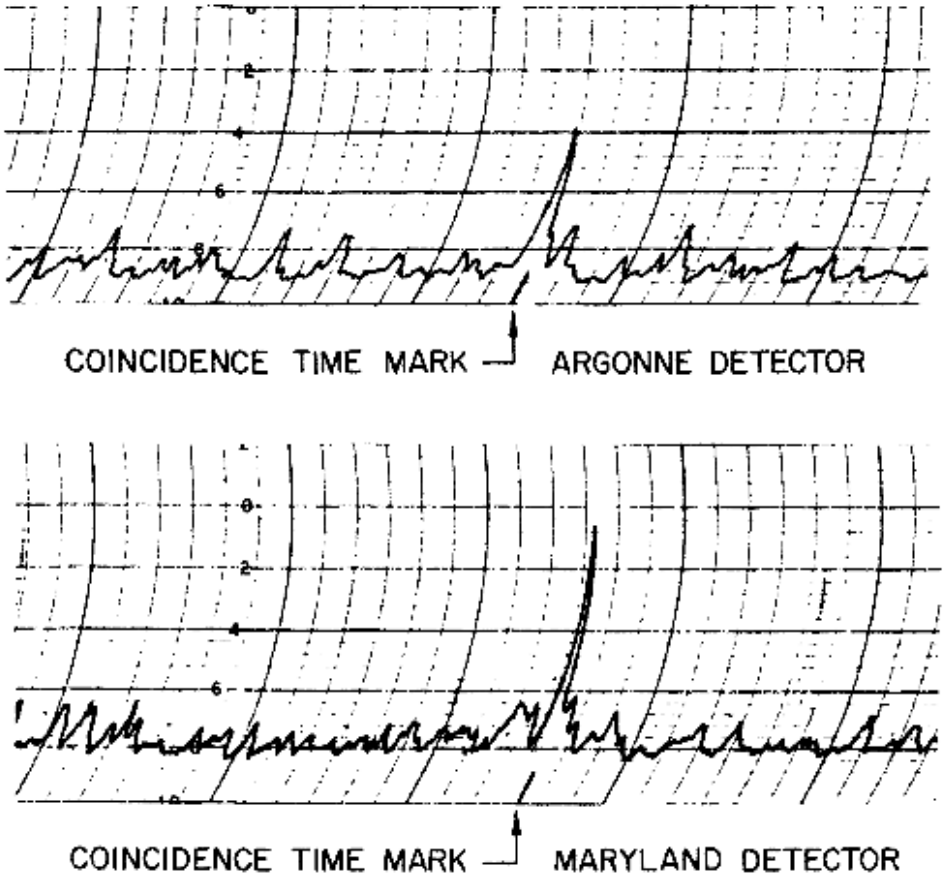
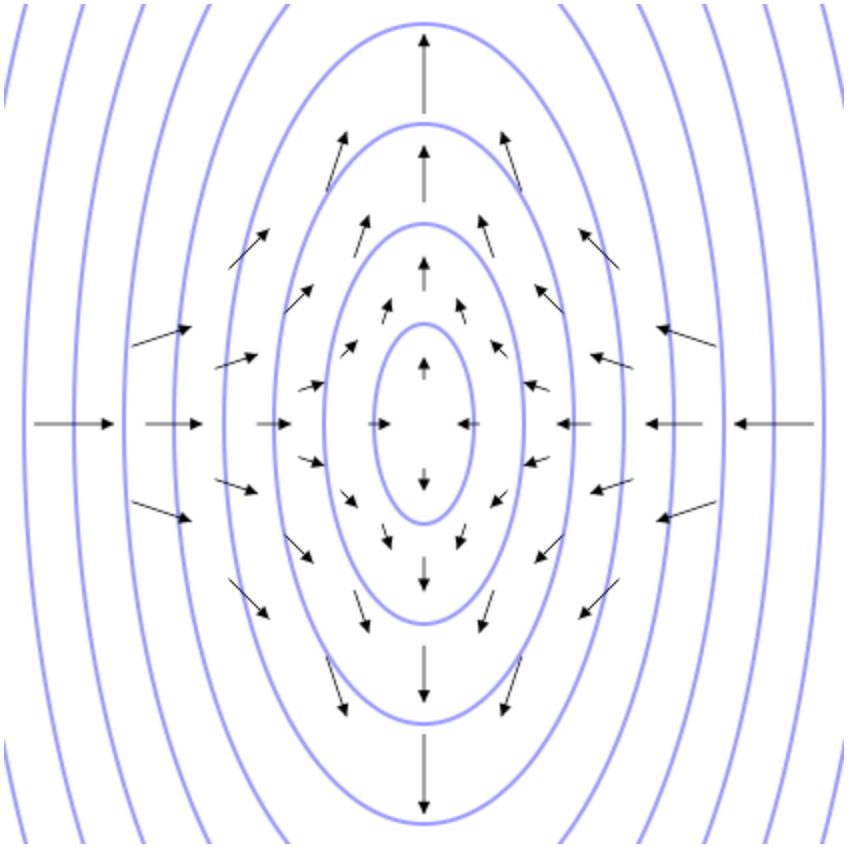
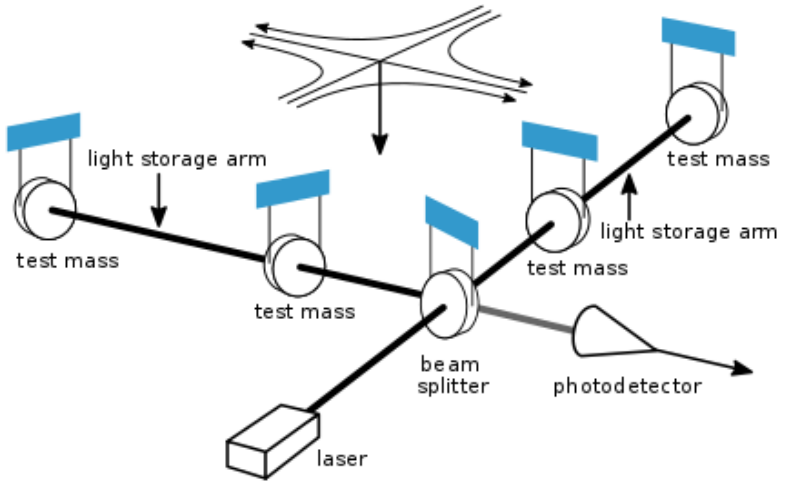


FIG. 2. Argonne National Laboratory and University of Maryland detector coincidence.

Stretching of space-time





Cosmological GWs

Amplification of gravitational waves in an isotropic universe

L. P. Grishchuk

State Astronomical Institute
(Submitted April 1, 1974)
Zh. Eksp. Teor. Fiz. **67**, 825–838 (September 1974)

It is shown that weak gravitational waves in a nonstationary isotropic universe can be amplified to a greater degree than indicated by the adiabatic law. It is a necessary (but not a sufficient) condition for the amplification that there should exist such a stage in the evolution of the universe when the characteristic time for change in the background metric is less than the period of the wave. In an expanding universe the wave is the more amplified the more strongly does the rate of evolution of the universe differ from the one which is dictated by matter with the equation of state $p = \epsilon/3$, and the earlier the wave had been "started." The superadiabatic amplification of gravitational waves denotes the possibility of creation of gravitons. An exceptional position is occupied by the "hot" isotropic universe with $p = \epsilon/3$, in which the superadiabatic amplification of gravitational waves and the production of gravitons is impossible. An estimate is made of the converse reaction of the gravitons on the background metric. Apparently, the production of gravitons forbids at least those of the iostropic singularities near which $p > \epsilon/3$.

Gravitational Waves from First-Order Cosmological Phase Transitions

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(Received 6 December 1991; revised manuscript received 26 May 1992)

A first-order cosmological phase transition that proceeds through the nucleation and collision of true-vacuum bubbles is a potent source of gravitational radiation. Possibilities for such include first-order inflation, grand-unified-theory-symmetry breaking, and electroweak-symmetry breaking. We have calculated gravity-wave production from the collision of two scalar-field vacuum bubbles, and, using an approximation based upon these results, from the collision of 20 to 30 vacuum bubbles. We present estimates of the relic background of gravitational waves produced by a first-order phase transition; in general, $\Omega_{GW} \sim 10^{-9}$ and $f \sim (10^{-6}\ \mathrm{Hz})(T/1\ \mathrm{GeV})$.

EW D VOLUME 45, NUMBER 12

Gravitational radiation from colliding vacuum bubbles

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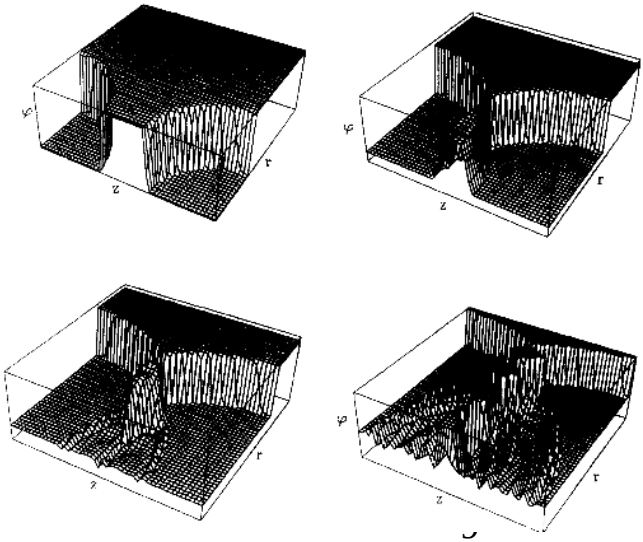
(Received 20 December 1991)



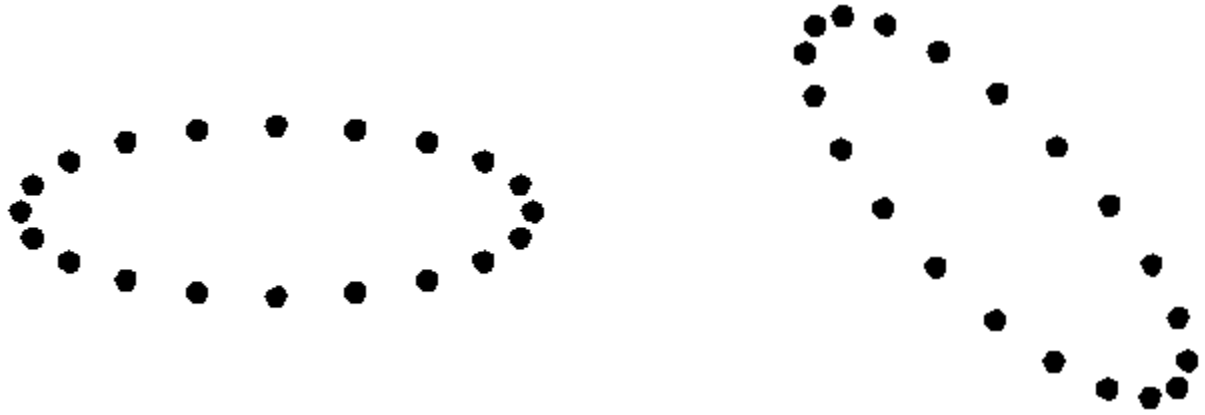
Why interesting if random?

- Energy spectrum
  - Peak(s)
  - Slopes
  - Relation to turbulence
- Polarization
  - Sign?
  - Relation magnetic helicity (=swirl of B field)

Solve for spatial part of h_{ij}

- Sourced by the stress tensor (Reynolds, Maxwell)
  - T_{ij} is symmetric tensor
  - 6 components
- Assume transverse $dT_{ij}/dx_j = 0$
  - 3 constraints
  - 3 components left
- Traceless
  - 1 more constraint
  - 2 components left

Correspond to + and x modes



$$\begin{pmatrix} \langle h_+(\boldsymbol{n}) h_+^*(\boldsymbol{n}') \rangle & \langle h_+(\boldsymbol{n}) h_\times^*(\boldsymbol{n}') \rangle \\ \langle h_+^*(\boldsymbol{n}) h_\times(\boldsymbol{n}') \rangle & \langle h_\times(\boldsymbol{n}) h_\times^*(\boldsymbol{n}') \rangle \end{pmatrix}$$

$$\frac{\delta_{drc}(\boldsymbol{n}-\boldsymbol{n}')}{4\pi} \begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}$$

Seto (2006)

State of the art: stochastic GW from

- MHD turbulence
  - Only analytic models
  - Using Lighthill approximation
- Expanding & colliding bubbles
  - Simulations by Hindmarsh et al. (2015)
  - Solve scalar field dynamics

PHYSICAL REVIEW D **76,** 083002 (2007)

Spectrum of gravitational radiation from primordial turbulence

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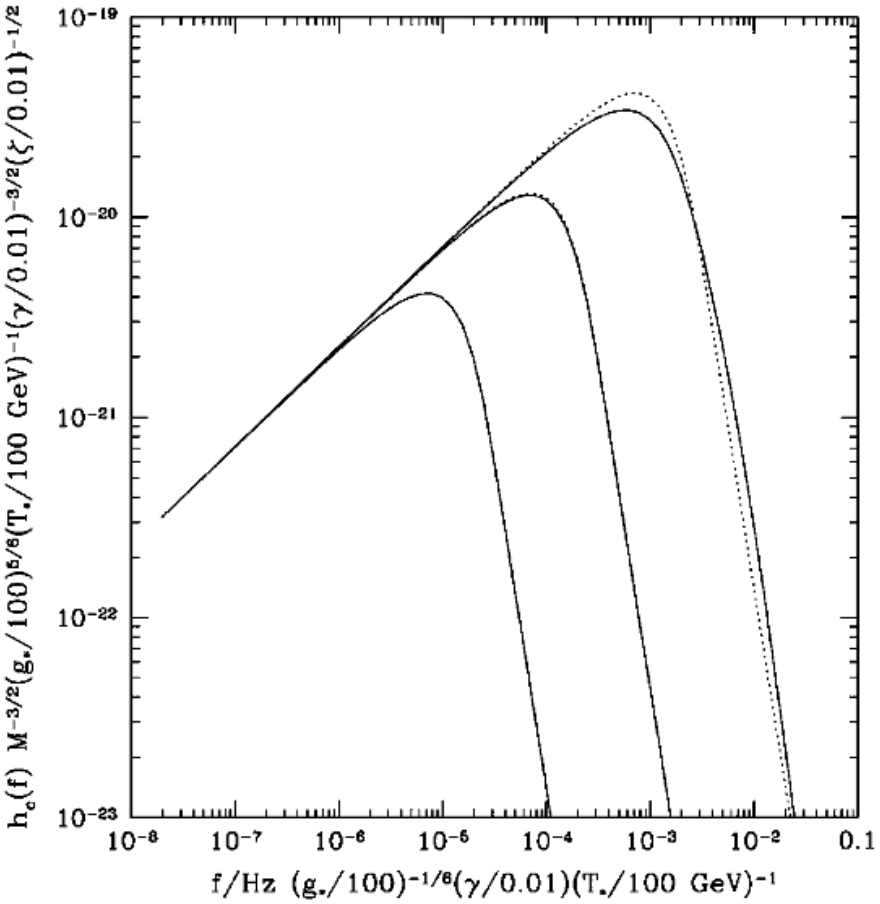
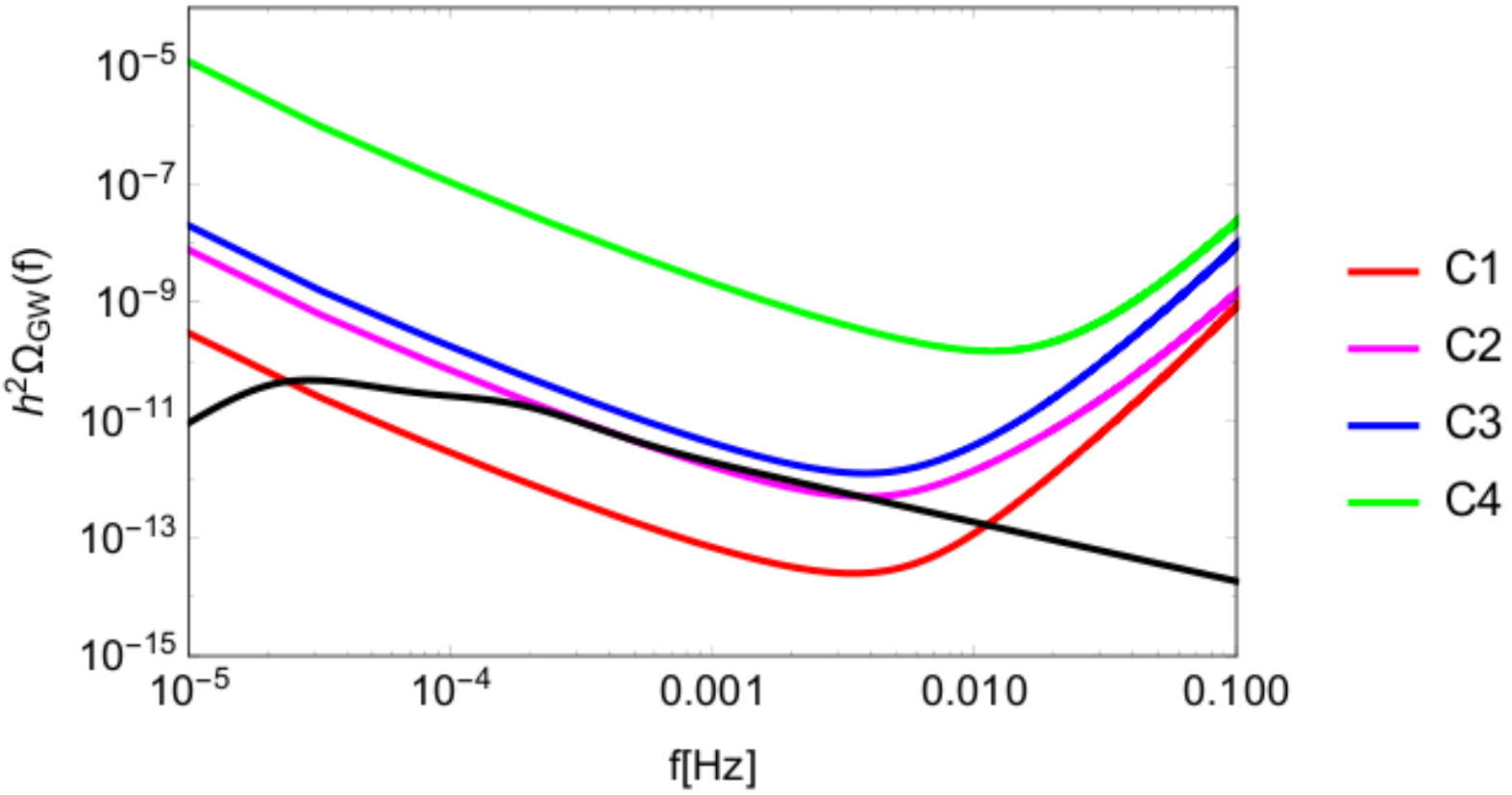
(Received 4 May 2007; published 5 October 2007)



- Kolmogorov turbulence
- Peak at f = 1 mHz
- Lighthill approx.
- $h(f) \sim 3\text{x}10^{-20}$
  - dimensionless

B-field from electroweak PT



- GW energy normalized by critical energy density
- Around 10^{-10} (nondimensional), at 0.03 mHz

Goal: compute GWs from real MHD turbulence

- Most popular scenario: electroweak phase transition
  - Weak and electromagnetic force decouple
- B-field from electroweak phase transition
  - Vachaspati (1991)
  - Cheng & Olinto (1994)
  - Baym, Bodeker, McLarran (1996)
- Time 10^{-11} sec
  - Horizon scale 0.3 cm
  - Now ~ 10AU → small
  - But turbulence → larger length scale (inverse cascade!)

Relativistic equations in expanding Universe

Energy momentum tensor

Brandenburg, Enqvist, Olesen
Phys Rev D 54, 1291 (1996)

$$T^{\mu\nu} = (p+\rho)U^{\mu}U^{\nu} + pg^{\mu\nu}$$
$$+ \frac{1}{4\pi}\left(F^{\mu\sigma}F^{\nu}{}_{\sigma} - \frac{1}{4}g^{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma} \right),$$

Conformal time, rescaled equations $\mathbf{S} = (p+\rho)\gamma^2\mathbf{v}$

$$\tilde{t} = \int dt/R. \qquad \tilde{\mathbf{S}} = R^4\mathbf{S}, \quad \tilde{p} = R^4 p, \quad \tilde{\rho} = R^4\rho, \quad \tilde{\mathbf{B}} = R^2\mathbf{B},$$
$$\tilde{\mathbf{J}} = R^3\mathbf{J}, \quad \text{and} \quad \tilde{\mathbf{E}} = R^2\mathbf{E}.$$

Equivalent to usual magneto-hydrodynamics

$$\frac{\partial\tilde{\mathbf{S}}}{\partial\tilde{t}} = -(\nabla\cdot\mathbf{v})\tilde{\mathbf{S}} - (\mathbf{v}\cdot\nabla)\tilde{\mathbf{S}} - \nabla\tilde{p} + \tilde{\mathbf{J}}\times\tilde{\mathbf{B}}.$$
$$\frac{\partial\tilde{\mathbf{B}}}{\partial\tilde{t}} = -\nabla\times\tilde{\mathbf{E}}, \quad \nabla\cdot\tilde{\mathbf{B}} = 0,$$

Small Lorentz factors, $\gamma \sim 1$

$$\frac{\partial \ln \tilde{\rho}}{\partial \tilde{t}} = -\frac{4}{3}(\mathbf{v}\cdot\boldsymbol{\nabla}\ln\tilde{\rho} + \boldsymbol{\nabla}\cdot\mathbf{v}) - \frac{\tilde{\mathbf{J}}\cdot\tilde{\mathbf{E}}}{\tilde{\rho}},$$

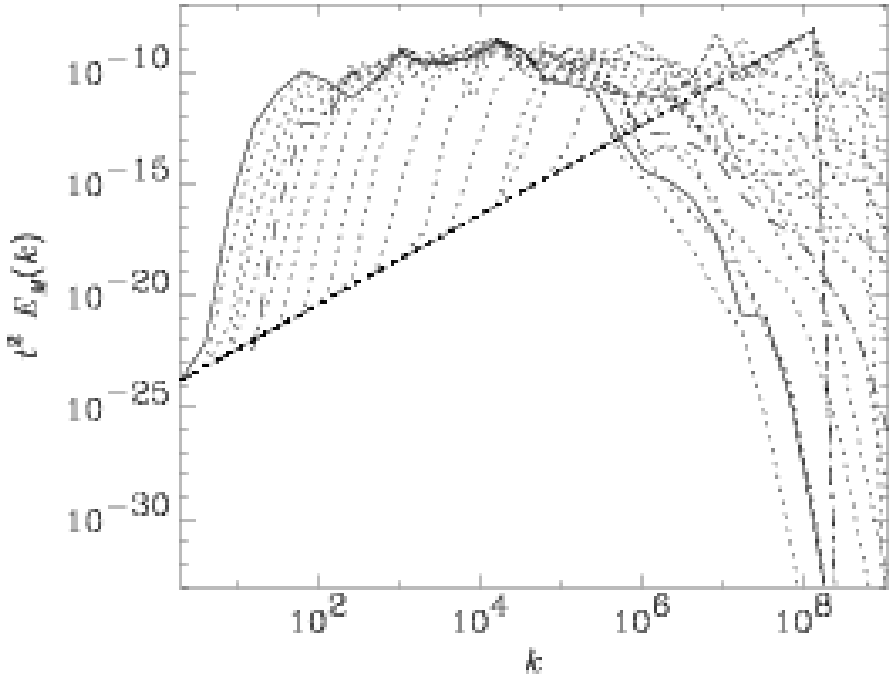
$$\frac{D\mathbf{v}}{D\tilde{t}} = -\mathbf{v}\left(\frac{D\ln\tilde{\rho}}{D\tilde{t}} + \boldsymbol{\nabla}\cdot\mathbf{v}\right) - \frac{1}{4}\boldsymbol{\nabla}\ln\tilde{\rho} + \frac{\tilde{\mathbf{J}}\times\tilde{\mathbf{B}}}{\frac{4}{3}\tilde{\rho}},$$

where $D/D\tilde{t} = \partial/\partial\tilde{t} + \mathbf{v}\cdot\boldsymbol{\nabla}$ is the total derivative, and

$$\frac{\partial\tilde{\mathbf{B}}}{\partial\tilde{t}} = \nabla\times(\mathbf{v}\times\tilde{\mathbf{B}}), \quad \tilde{\mathbf{J}} = \boldsymbol{\nabla}\times\tilde{\mathbf{B}}.$$

Magnetohydrodynamic turbulence

The conclusion from the above expressions is thus that *the MHD equations in an expanding universe with zero curvature are the same as the relativistic MHD equations in a nonexpanding universe, provided the dynamical quantities are replaced by the scaled ''tilde'' variables, and provided conformal time* $\tilde{t}$ *is used.* The effect of this is, as usual, that



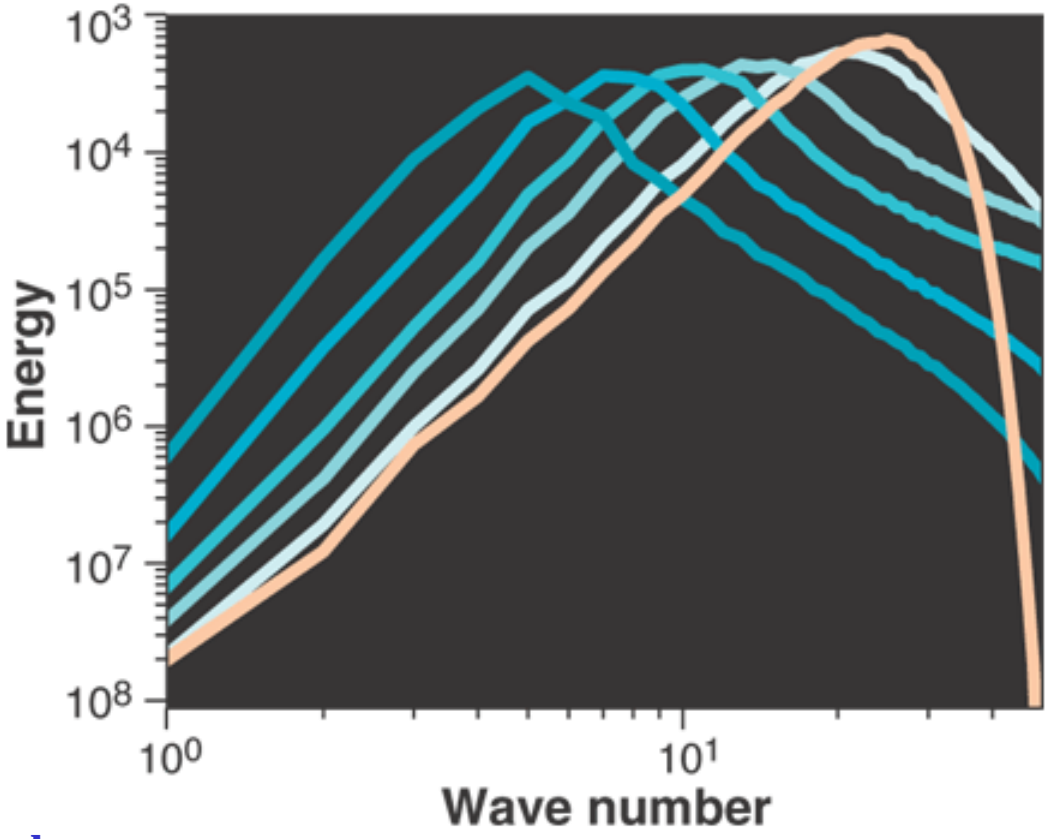
shell models $\quad \dfrac{db_n}{d\tilde{t}} = M_n(v,b)$

$$M_n(v,b) = ik_n(A-C)(v^*_{n+1}b^*_{n+2} - b^*_{n+1}v^*_{n+2})$$
$$+ ik_n(B+\tfrac{1}{2}C)(v^*_{n-1}b^*_{n+1} - b^*_{n-1}v^*_{n+1})$$
$$- ik_n(\tfrac{1}{2}B-\tfrac{1}{4}A)(v^*_{n-2}b^*_{n-1} - b^*_{n-2}v^*_{n-1}),$$

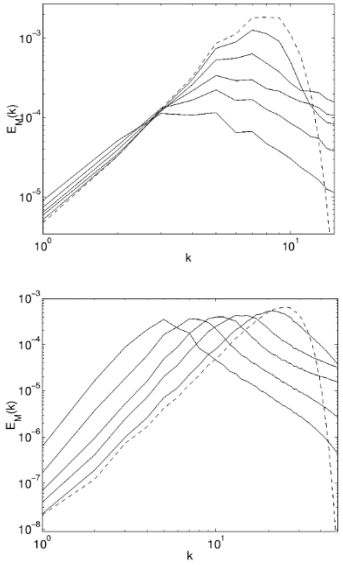
$$\frac{4}{3}\rho_0 \frac{dv_n}{d\tilde{t}} = N_n(v,b)$$

3-D decay simulations

Initial slope
$E \sim k^4$



helical vs nonhelical



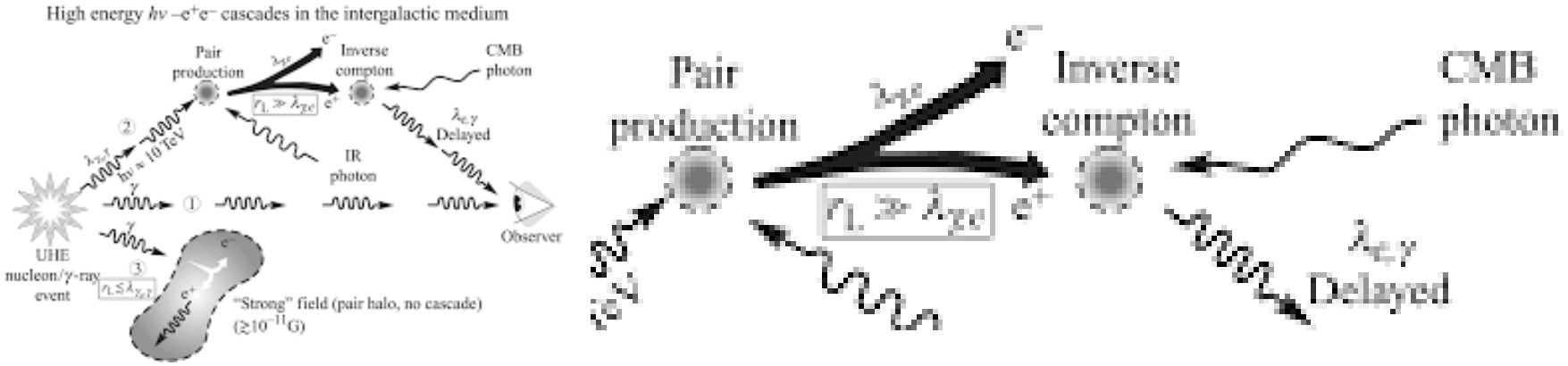
Christensson et al.
(2001, PRE 64, 056405)

Evidence for Strong Extragalactic Magnetic Fields from Fermi Observations of TeV Blazars

Andrii Neronov* and Ievgen Vovk

Science 328, 73 – 75 (2010)

Magnetic fields in galaxies are produced via the amplification of seed magnetic fields of unknown nature. The seed fields, which might exist in their initial form in the intergalactic medium, were never detected. We report a lower bound $B \geq 3 \times 10^{-16}$ gauss on the strength of intergalactic magnetic fields, which stems from the nonobservation of GeV gamma-ray emission from electromagnetic cascade initiated by tera–electron volt gamma rays in intergalactic medium. The bound improves as $\lambda_B^{-1/2}$ if magnetic field correlation length, λ_B, is much smaller than a megaparsec. This lower bound constrains models for the origin of cosmic magnetic fields.



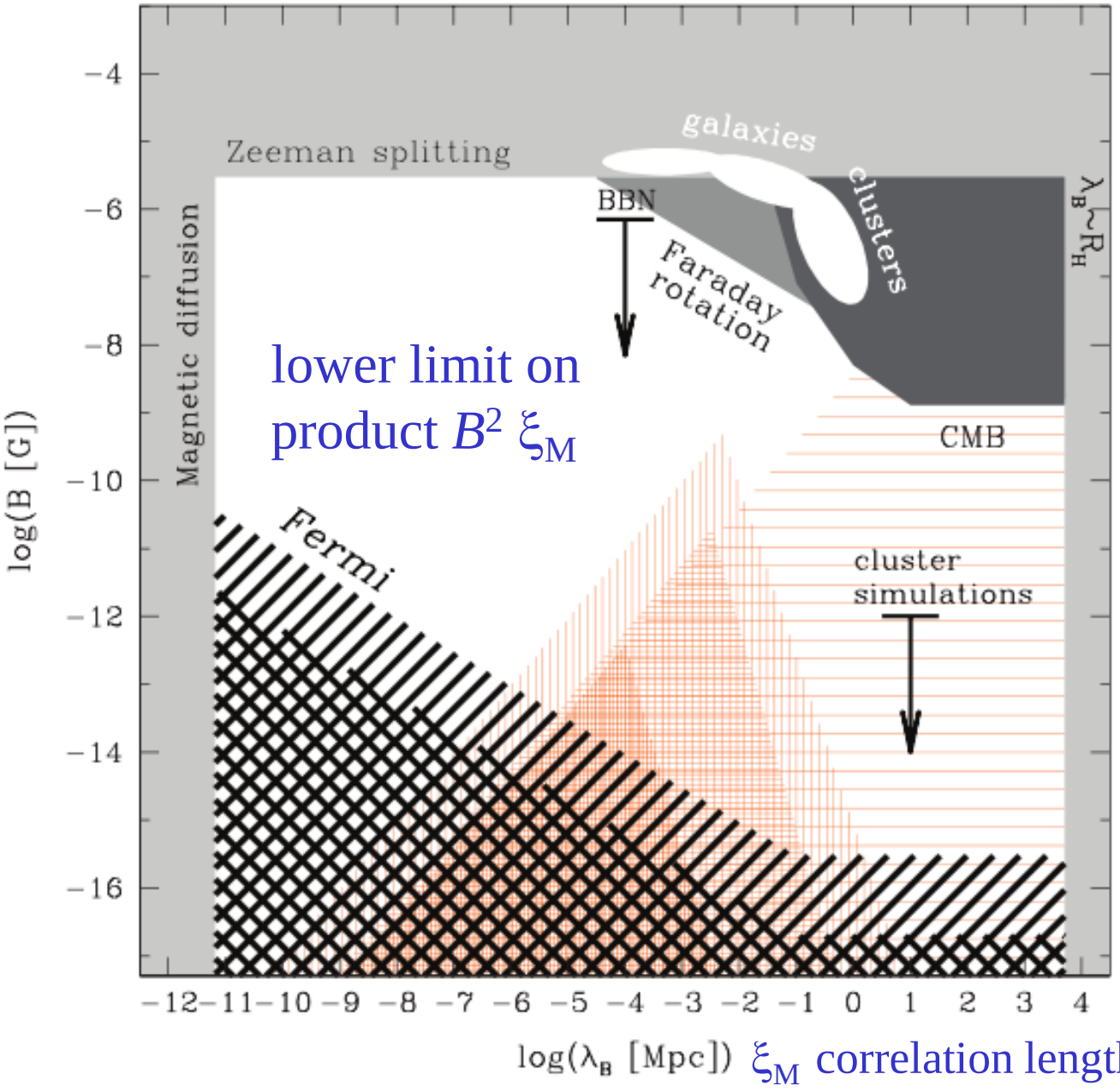
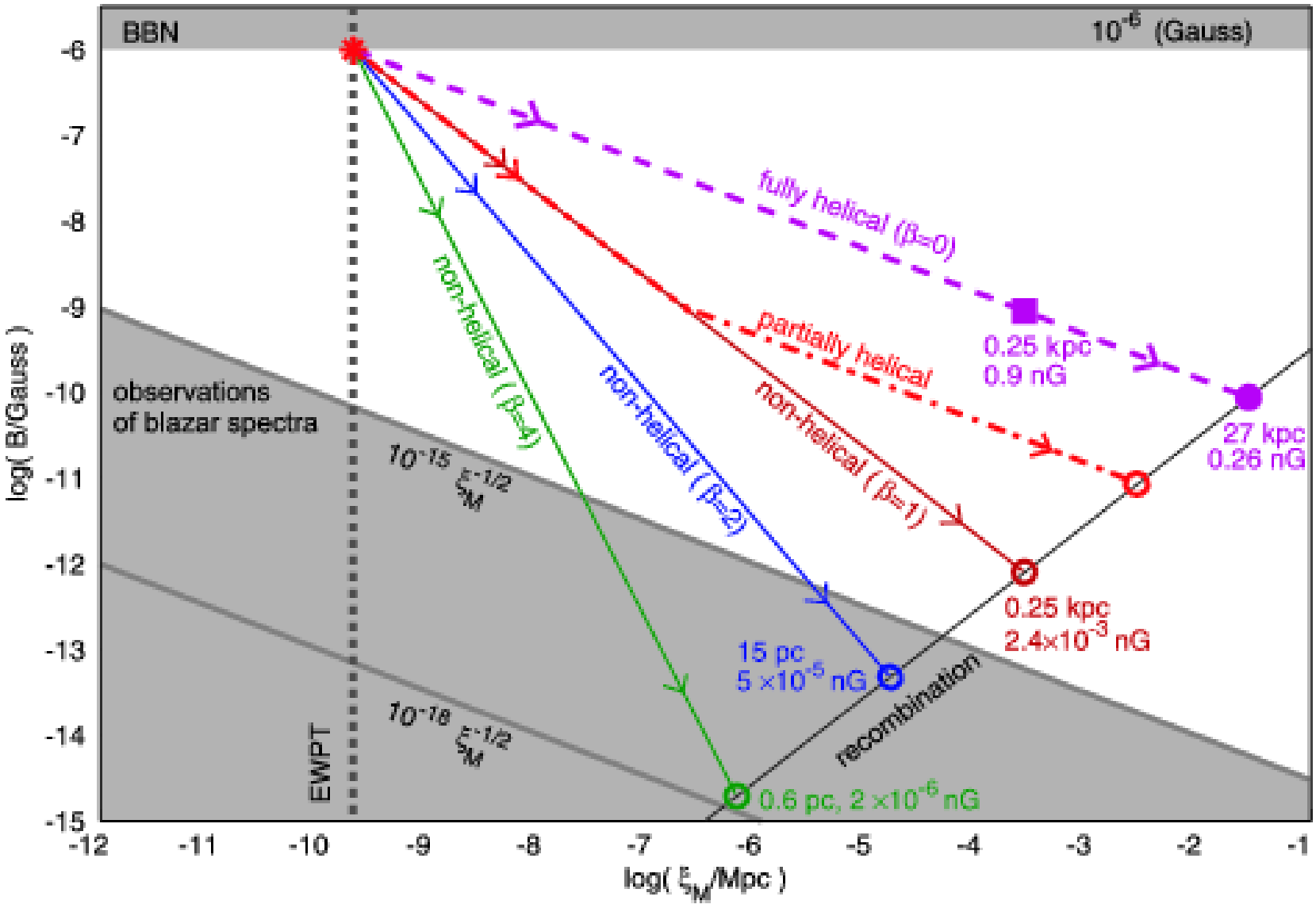


Fig. 2. Light, medium, and dark gray: known observational bounds on the strength and correlation length of EGMF, summarized in (*25*). The bound from Big Bang nucleosynthesis (BBN) is from (*2*). The black hatched region shows the lower bound on the EGMF derived from observations of 1ES 0347-121 (cross-hatching) and 1ES 0229+200 (single diagonal hatching) in this paper. Orange hatched regions show the allowed ranges of B and λ_B for magnetic fields generated at the epoch of inflation (horizontal hatching), the electroweak phase transition (dense vertical hatching), QCD phase transition (medium vertical hatching), and epoch of recombination (light vertical hatching) (*25*). White ellipses show the range of measured magnetic field strengths and correlation lengths in galaxies and galaxy clusters.

Turb decay: B decreases, ξ_M increases

BBN = Big-bang nucleosynthesis



Simple example

$$\left(\partial_t^2 + 3H\partial_t - c^2\nabla^2\right) h_{ij}(\boldsymbol{x},t) = \frac{16\pi G}{c^2}\, T_{ij}^{\mathrm{TT}}(\boldsymbol{x},t)$$

$$T_{ij}(\boldsymbol{x},t) = \rho u_i u_j - B_i B_j + (\boldsymbol{B}^2/2 + p)\delta_{ij}$$

Example

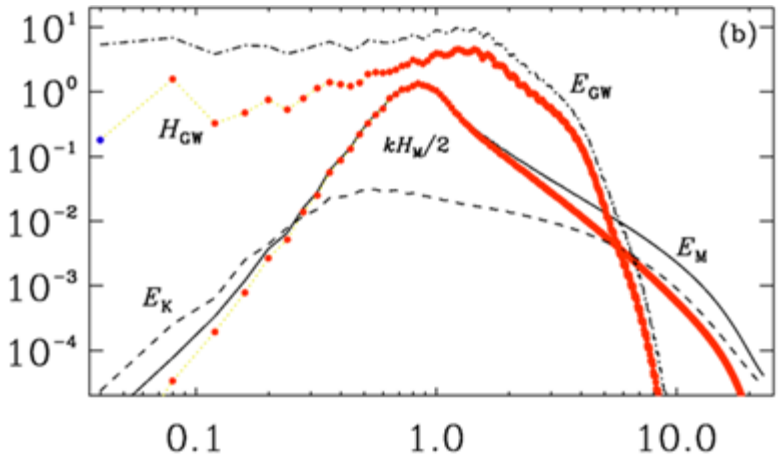
$$\boldsymbol{B}(x) = B_0 \begin{pmatrix} 0 \\ \sigma \sin kx \\ \cos kx \end{pmatrix}$$

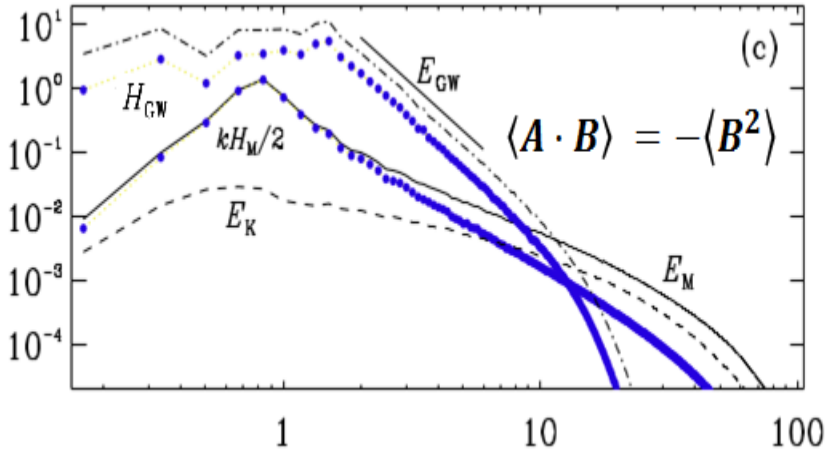
Traceless-transverse

$$T_{ij}(x) = \mathcal{E}_{\mathrm{M}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\cos 2kx & \sigma \sin 2kx \\ 0 & \sigma \sin 2kx & \cos 2kx \end{pmatrix}$$

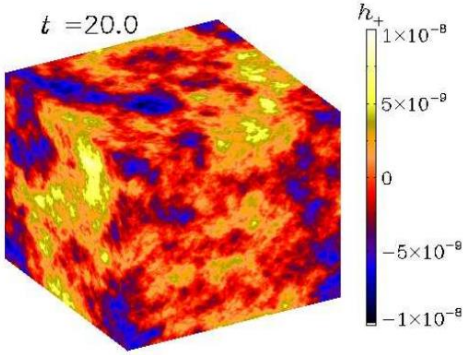
Early examples

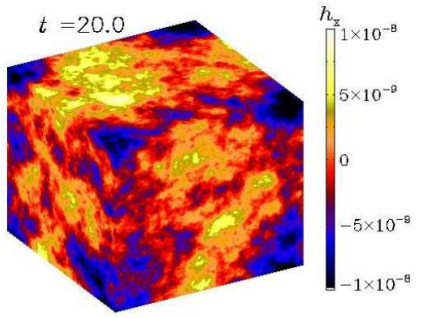
Projection operator $\hat{T}_{ij}^{\rm TT}(\boldsymbol{k},t) = (P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm})\hat{T}_{lm}(\boldsymbol{k},t)$



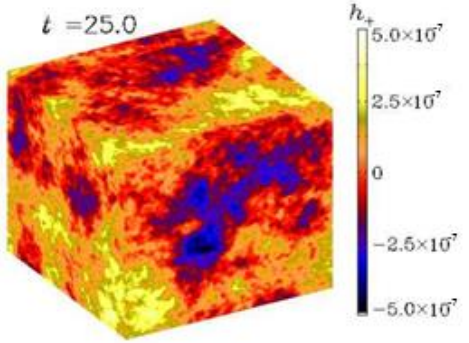


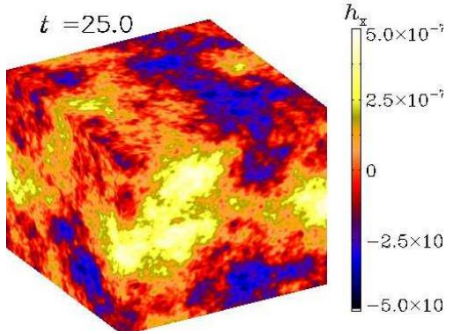
- Magnetic energy spectrum $\int E_{\rm M}(k,t)\,dk = \langle \boldsymbol{B}^2\rangle/2$
- Positive helicity (red), negative (blue)
- GW energy spectra



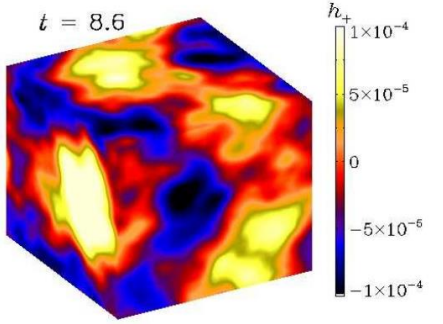


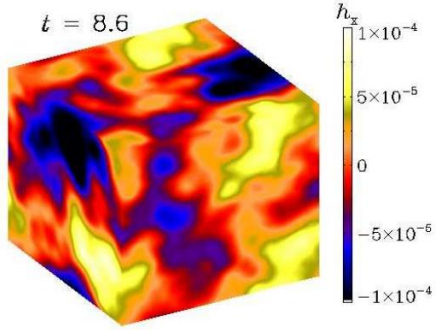
$k_M/k_H = 300$





$k_M/k_H = 60$





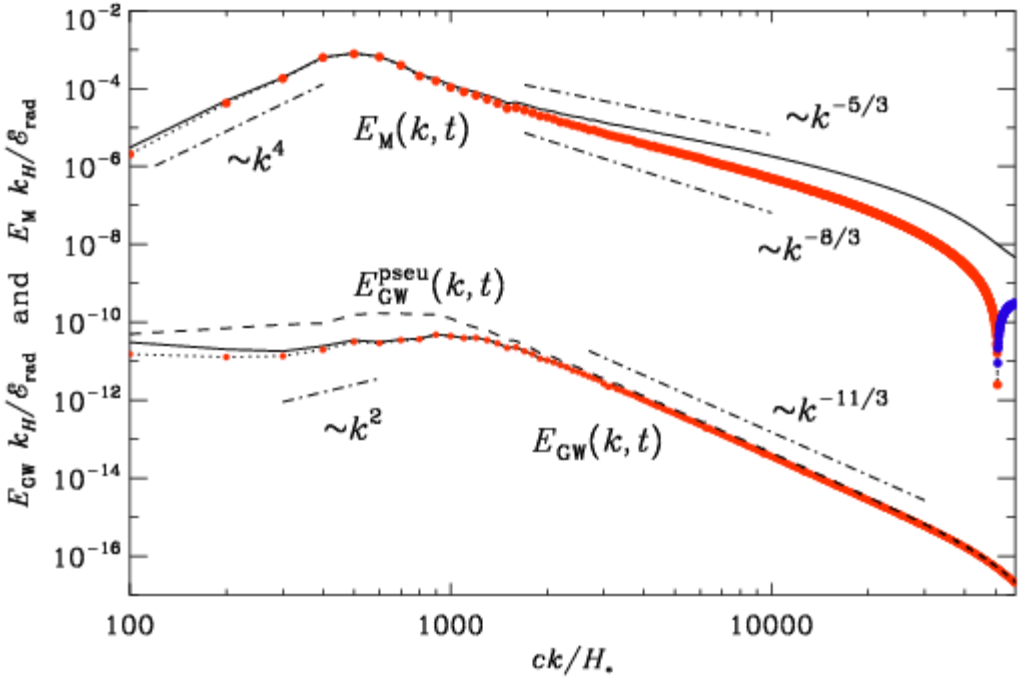
$k_M/k_H = 2$

Treatment in Pencil Code

Scale factor a → $\tilde{h}_{ij} = a h_{ij}$ …but then drop tilde

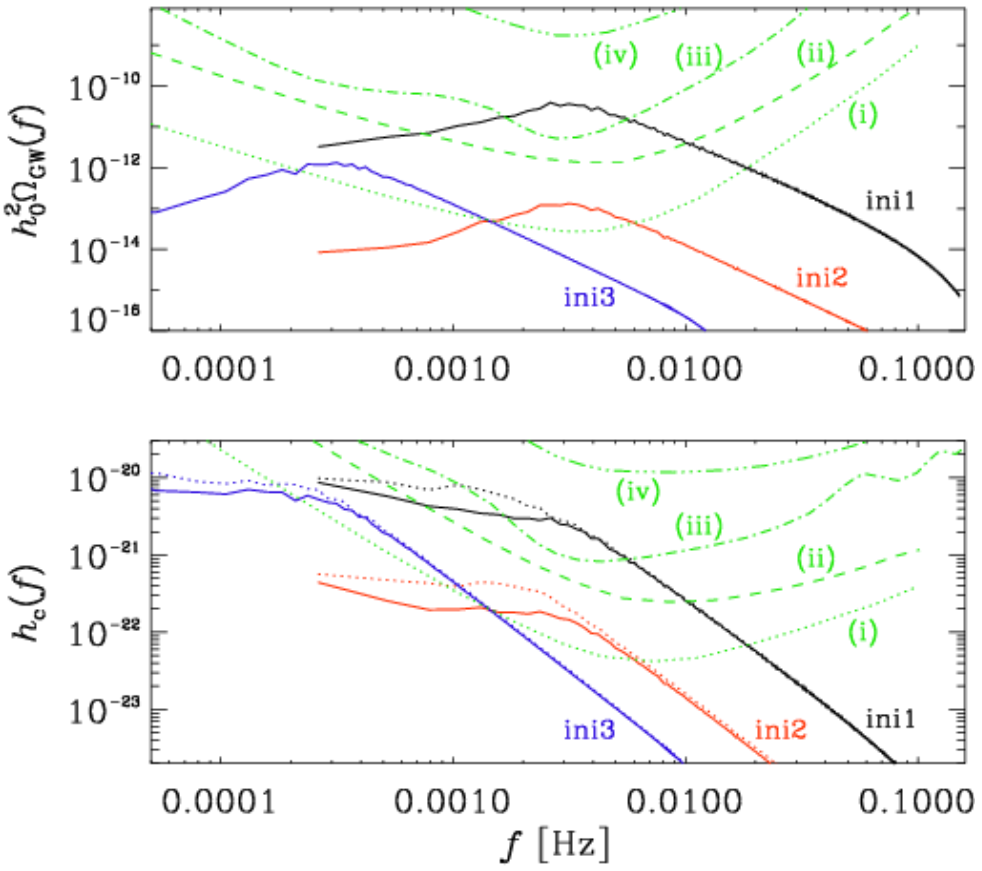
$$\left(\partial_{\bar{t}}^2 - c^2\nabla^2\right) h_{ij}(\boldsymbol{x},\bar{t}) = 6\, T_{ij}^{\rm TT}(\boldsymbol{x},\bar{t})/\bar{t}$$

- No damping
- Except of the source
- Kolmogorov
- GW spectrum by k^2 steeper



Normalization

$$h_0^2 \Omega_{\rm GW}(k,t) = k S_h(k,t)/(6\, H_0^2 a_0^4)$$

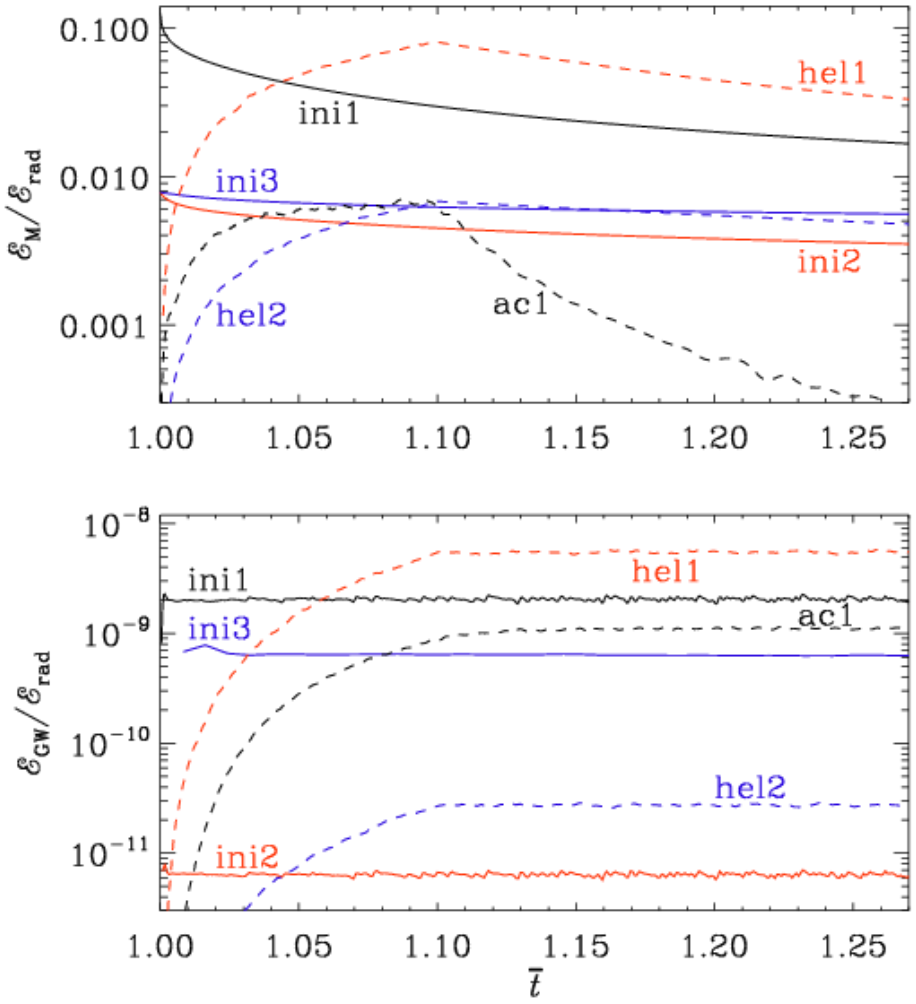


$$E_{\rm GW}(k,t) = c^2 S_h(k,t)/(16\pi G)$$

$$\mathcal{E}_{\rm crit} = 3H_0^2 c^2/8\pi G$$

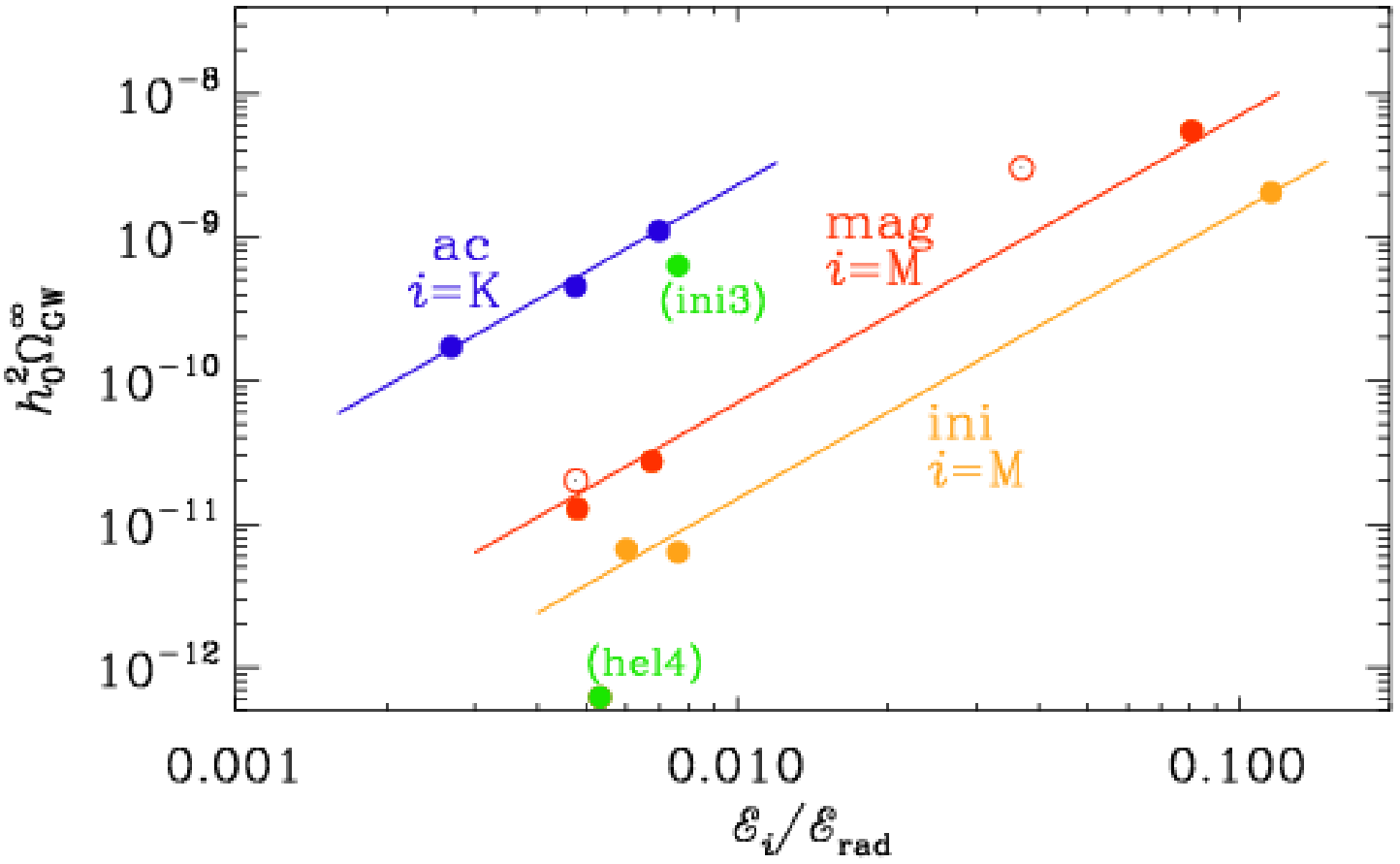
- All runs with sudden magn. field initialization
- Diff init mag energy
- Different peak *k*

Magnetic & GW energies



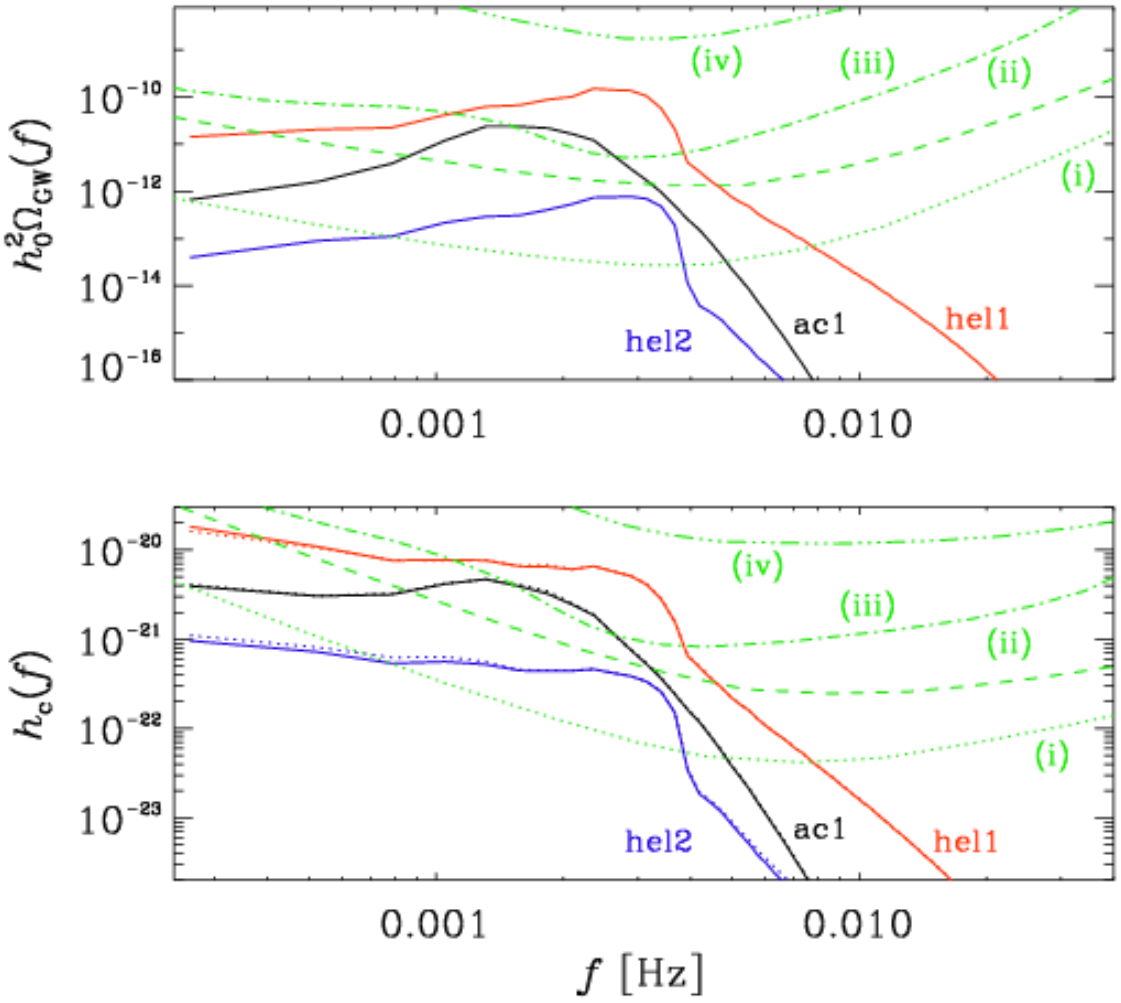
- Ini1-3 with initial field: only decay
- GW energy const
- Hel1+2 and ac1
- Driving until t=1.1
- GW energy builds up, → then const
- GW energy quadratic in ***B***-field

Quadratic scaling



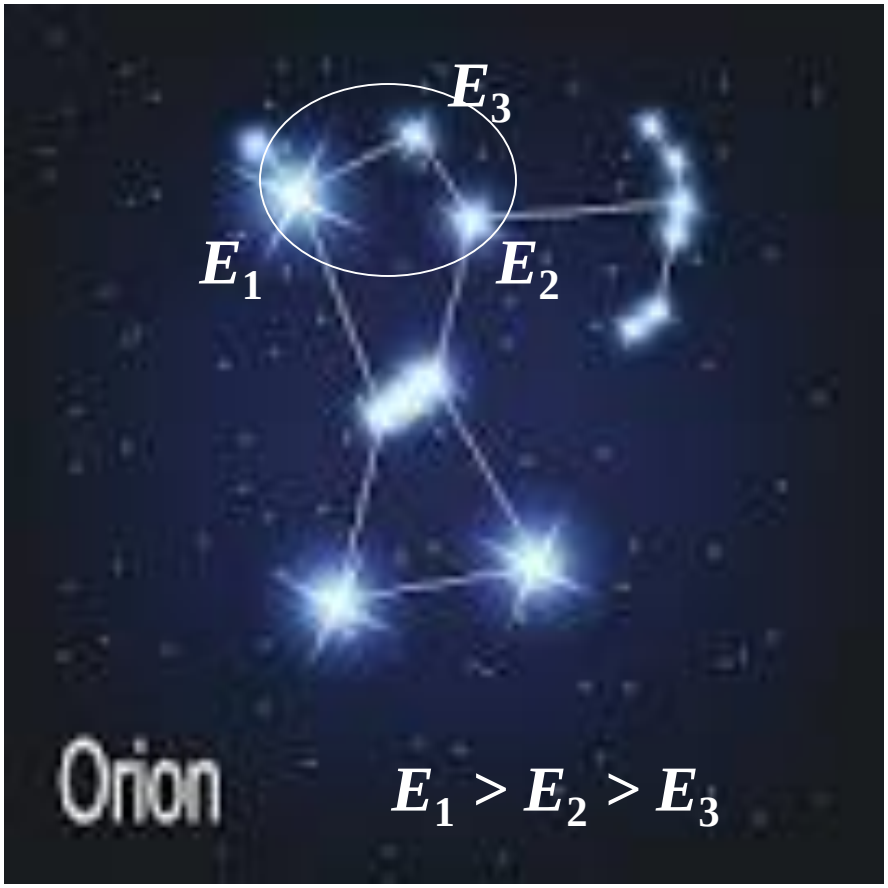
- Acoustic driving → strongest GW field
- Quadratic scaling

Spectra from driven case



- Sharp drop
- Long subinertial range

Other measures of global helicity?



Method 1

What is the sign of

$$(\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{n}_3$$

$$\mathbf{n} = \begin{pmatrix} \cos\alpha \sin\delta \\ \cos\alpha \cos\delta \\ \sin\alpha \end{pmatrix}$$

Compute this for all triples

Search for CP violating signature of intergalactic magnetic helicity in the gamma-ray sky

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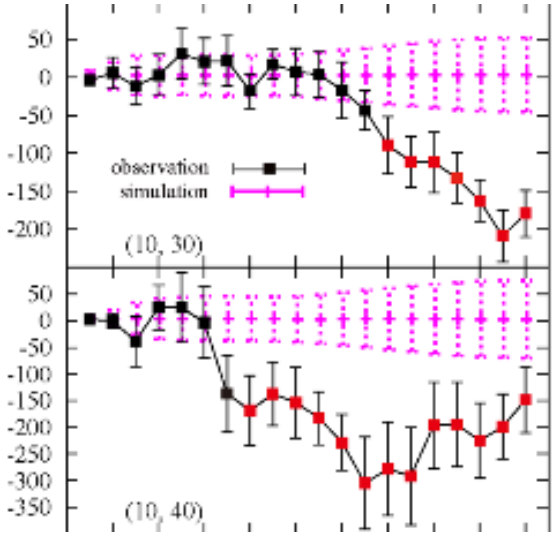


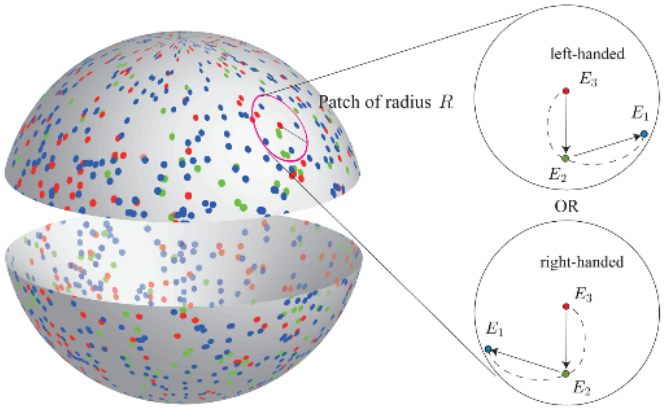


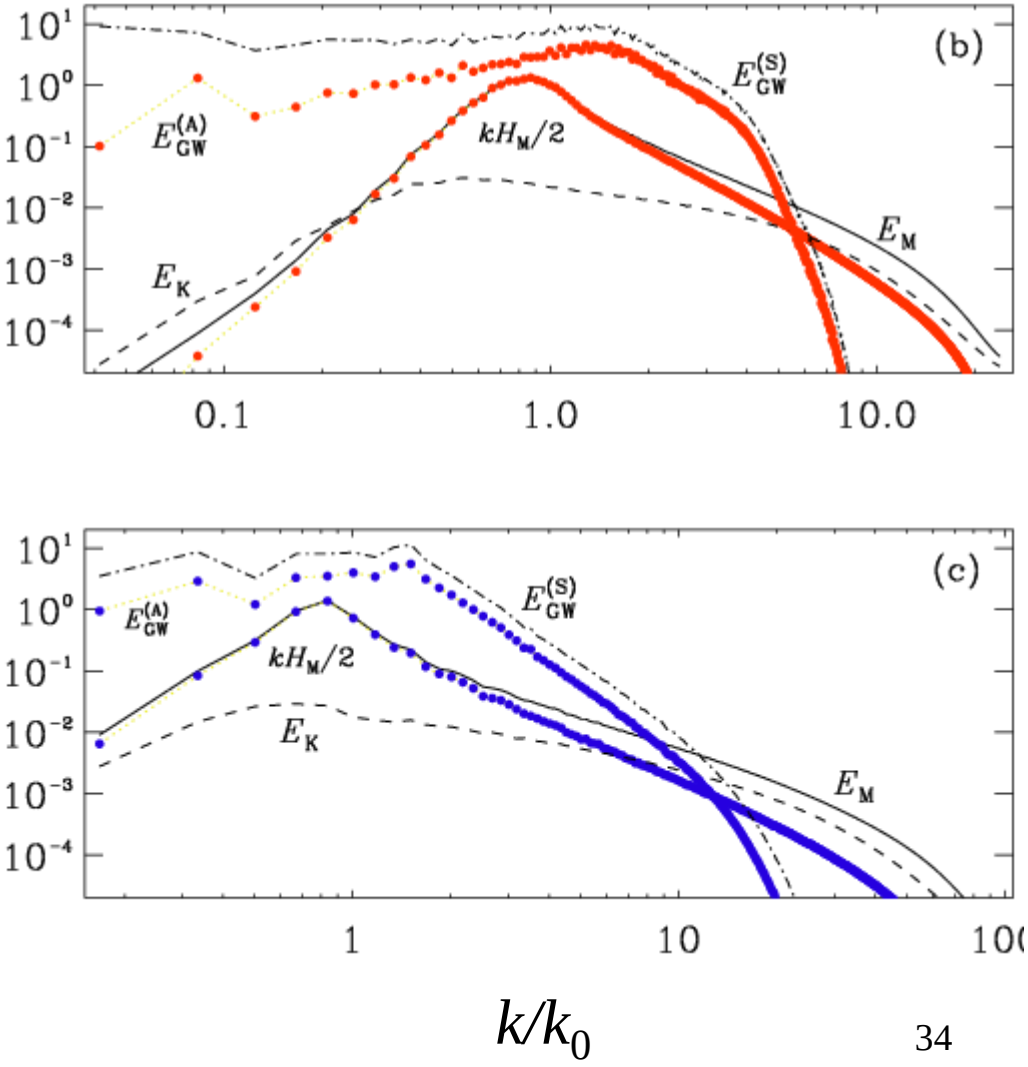
Figure 2. Illustration of the cut-sky with gamma-rays distributed on it. Patches of radius R degrees are centred on the highest energy gamma-rays. In those patches we test if the lower energy photons are distributed along left- or right-handed spirals.

Finally, since we find $Q < 0$, this indicates that the cosmological magnetic field has left-handed helicity. This could be very interesting for particle physics and early universe cosmology since baryogenesis, which requires fundamental CP violation, predicts magnetic

Helicity from gravitational waves

Method 2

- GW circularly polarized ← hel. (Kahniashvili+05)
- GW background noise w/ eLISA
  - Red: pos hel or pos circ pol
  - Blue: negative



Conclusion

- Early universe: decaying turbulence
  - Self-similar decay
- GW excited
  - Can be detected with LISA (if lucky)
  - Polarization: sign of parity violation
- Mirror asymmetry can be measured
  - Photon arrival directions
  - EB polarization (from Q,U → parity even/odd)
  - Gravitational waves