# Vorticity production

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# Vorticity production

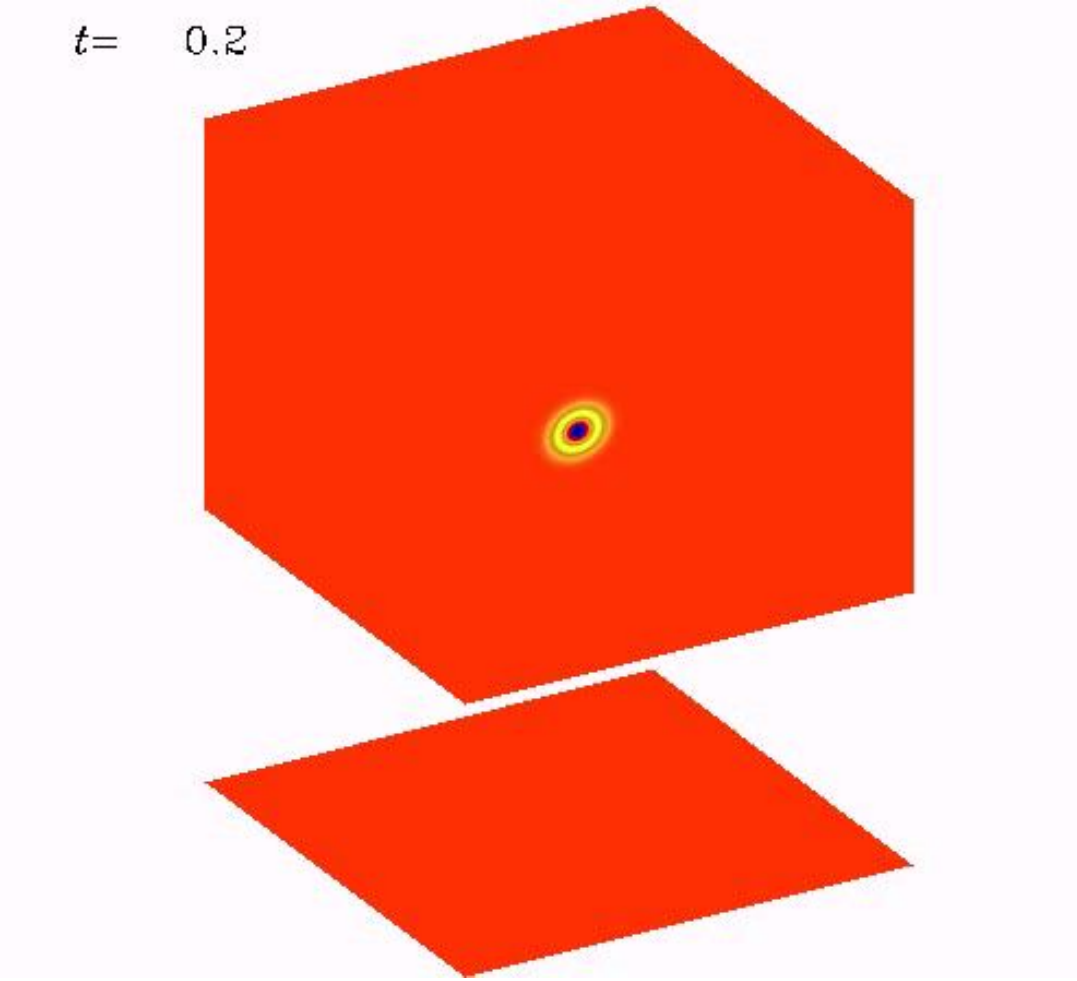
- Forcing purely potential, where does $\omega_{rms}$ come from?
- Sort of small-scale vortex-dynamo problem

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{u} \times \mathbf{B} \right) + \eta \nabla^2 \mathbf{B}$$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times \left( \mathbf{u} \times \boldsymbol{\omega} \right) + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times \left( 2\nu \mathbf{S} \cdot \nabla \ln \rho \right)$$

- One source is *always* viscosity
- Other sources: rotation, shear, baroclinicity, *B*-field

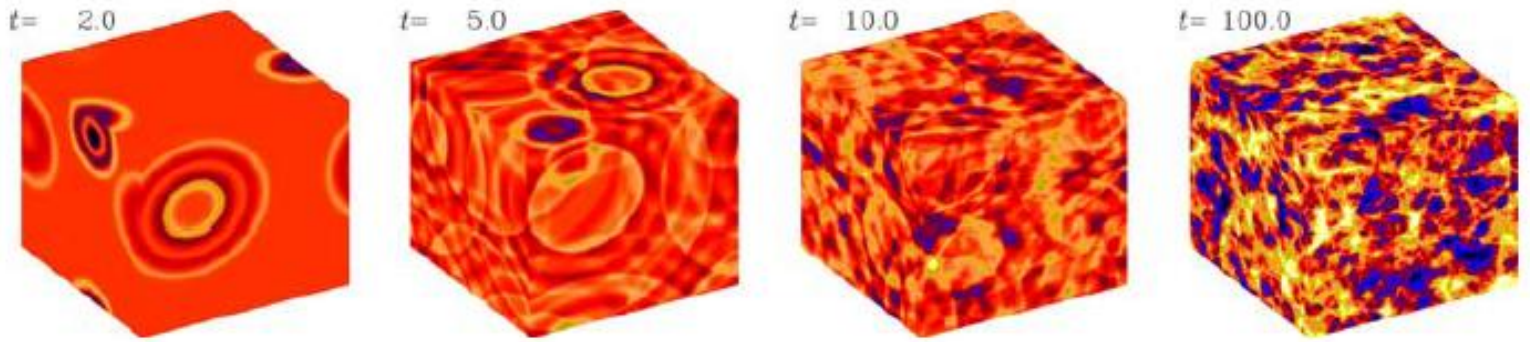
# Gaussian expansion waves

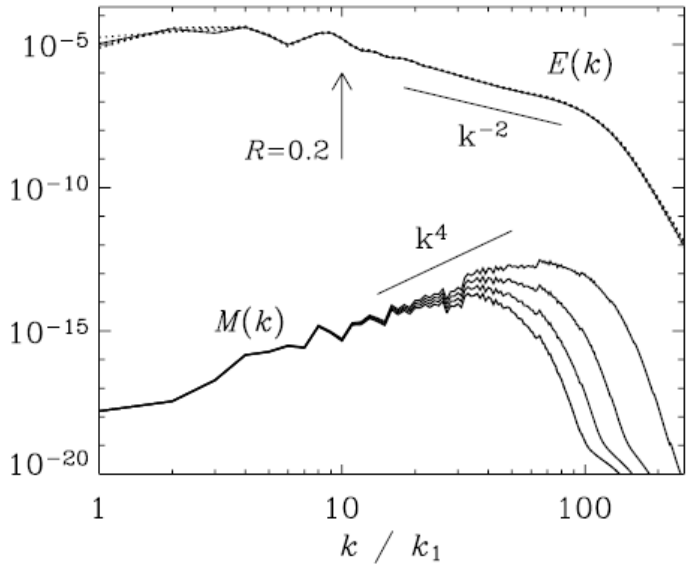



Motivated by SN explosions

Mee & Brandenburg (2006)

# No dynamo from potential flows

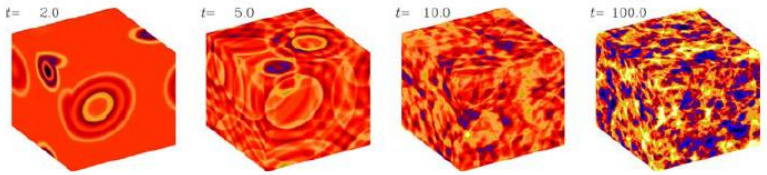



No dynamo action in nearly potential flows (at least not fo far)

# No vorticity either (small Ma)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

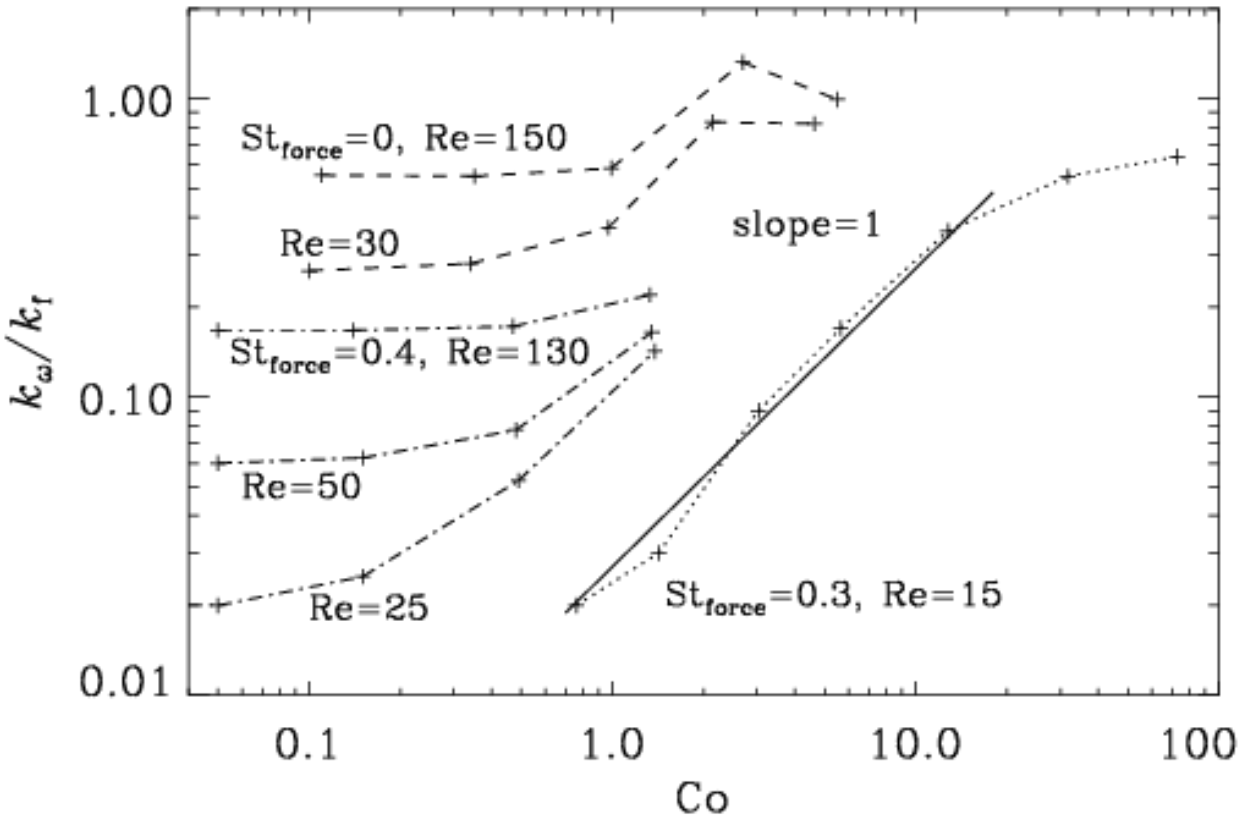



$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times (2\nu \mathbf{S} \cdot \nabla \ln \rho)$$

**Table 1.** Results for the normalized vorticity, $\omega_{\rm rms}/(u_{\rm rms}\, k_{\rm peak})$, as a function of Re and resolution. The durations of the runs vary between $N_{\rm turn} = 16$ and 250 turnover times, and $\nu$ varies between $10^{-3}$ and $5 \times 10^{-5}$. For the high resolution run with $512^3$ mesh points and $\nu = 5 \times 10^{-5}$, we have $N_{\rm turn} = 59$ turnover times.

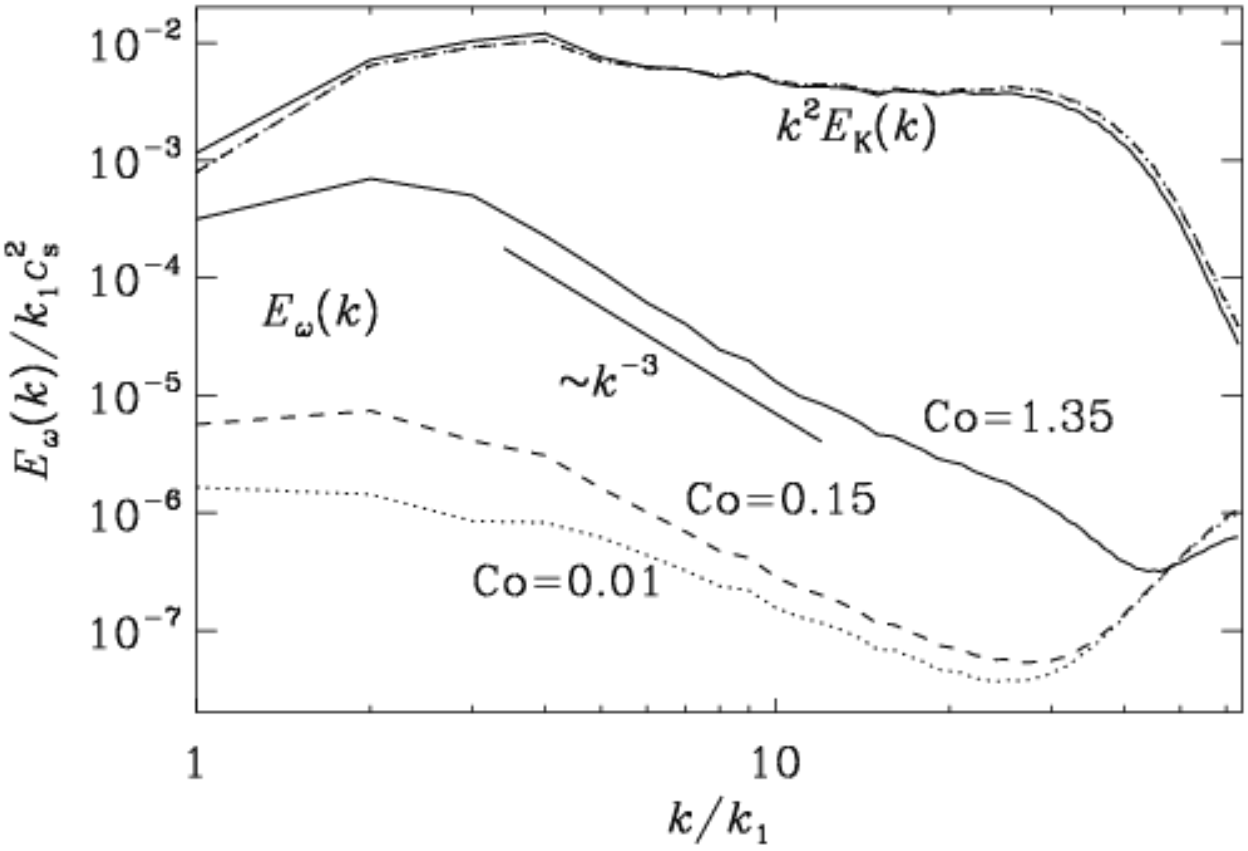
| Re | $512^3$ | $256^3$ | $128^3$ |
|---|---|---|---|
| 50 | $1.4 \times 10^{-3}$ | $8.7 \times 10^{-3}$ | |
| 25 | | $1.1 \times 10^{-2}$ | $2.9 \times 10^{-3}$ |
| 12 | | $1.6 \times 10^{-2}$ | $2.0 \times 10^{-3}$ |
| 4 | | $7.6 \times 10^{-3}$ | $1.5 \times 10^{-3}$ |

# Linear increase with *rotation*



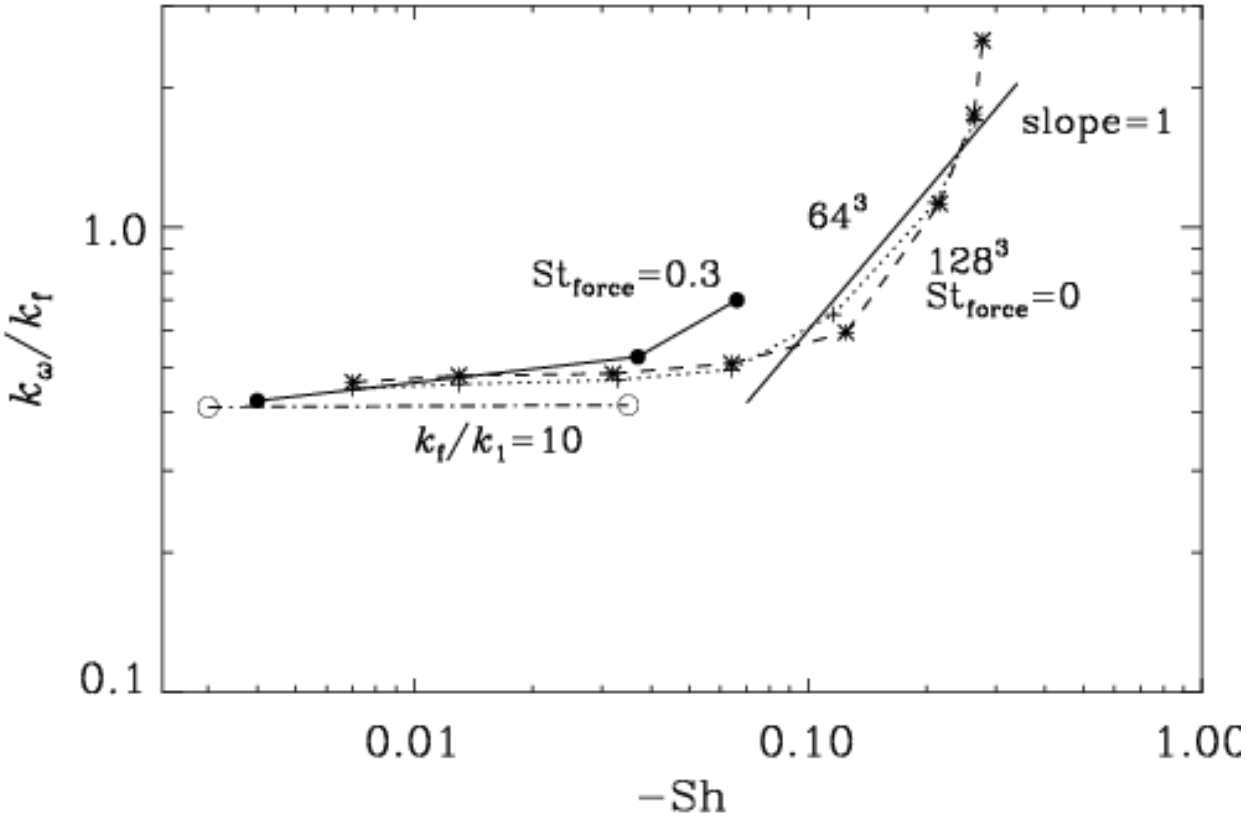

$$\omega_{rms} = 2\Omega\tau\left\langle \left(\nabla_\perp \cdot \mathbf{u}_\perp\right)^2 + \left(\nabla_z \mathbf{u}_\perp\right)^2 \right\rangle^{1/2}$$

# Clearly a large-scale effect




Small-scale effect independent of Ω, but perhaps spurious?

# Similar growth with *shear* rate

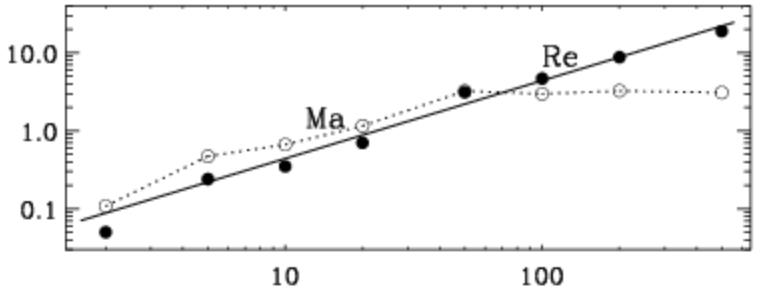


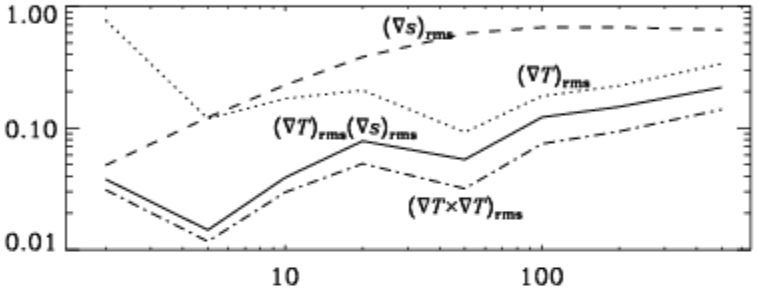

Spurious because of shearing sheet boundary conditions?

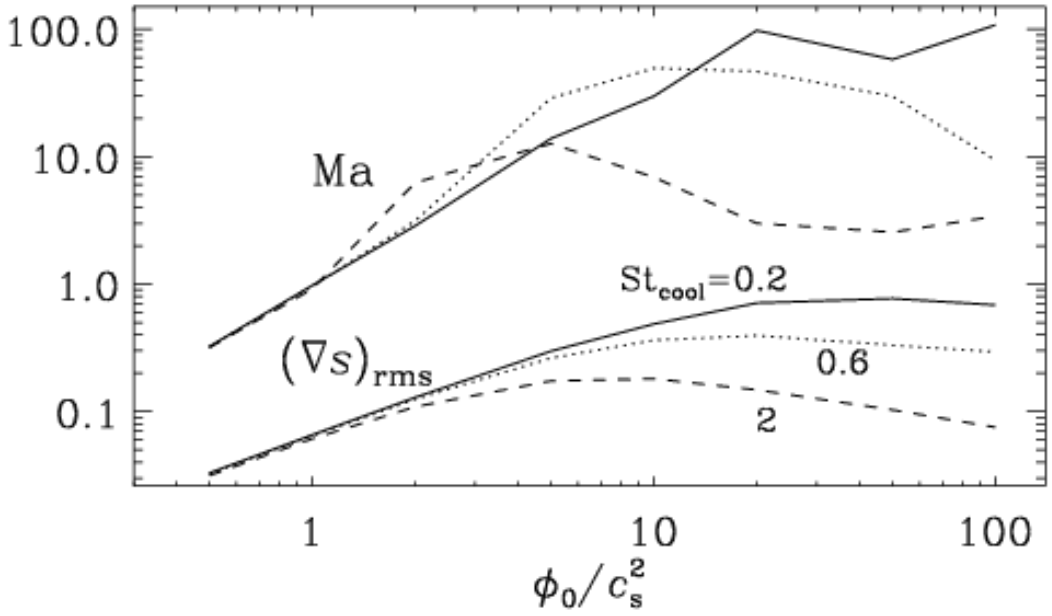
# *Baroclinic* effect

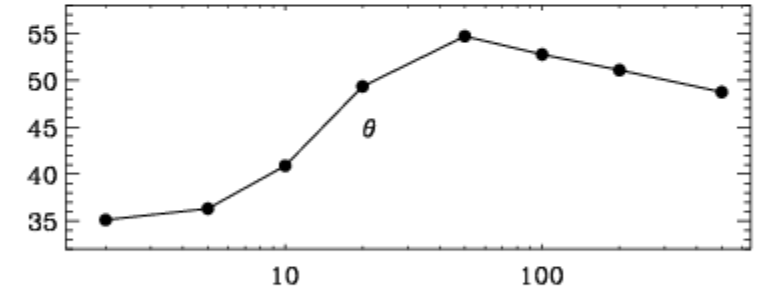
$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \ldots + \nabla \times \left( -\nabla T + T \nabla s \right)$$

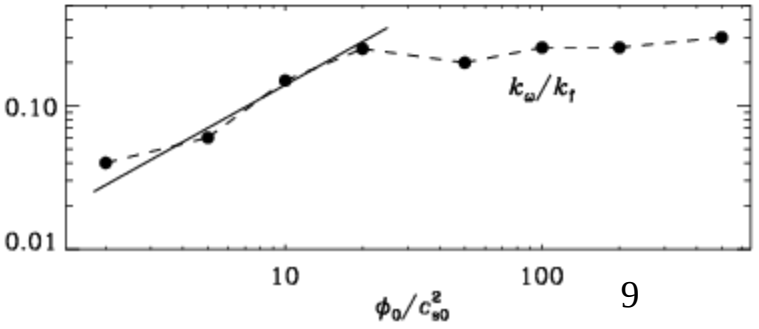
$$= \ldots + \nabla T \times \nabla s$$



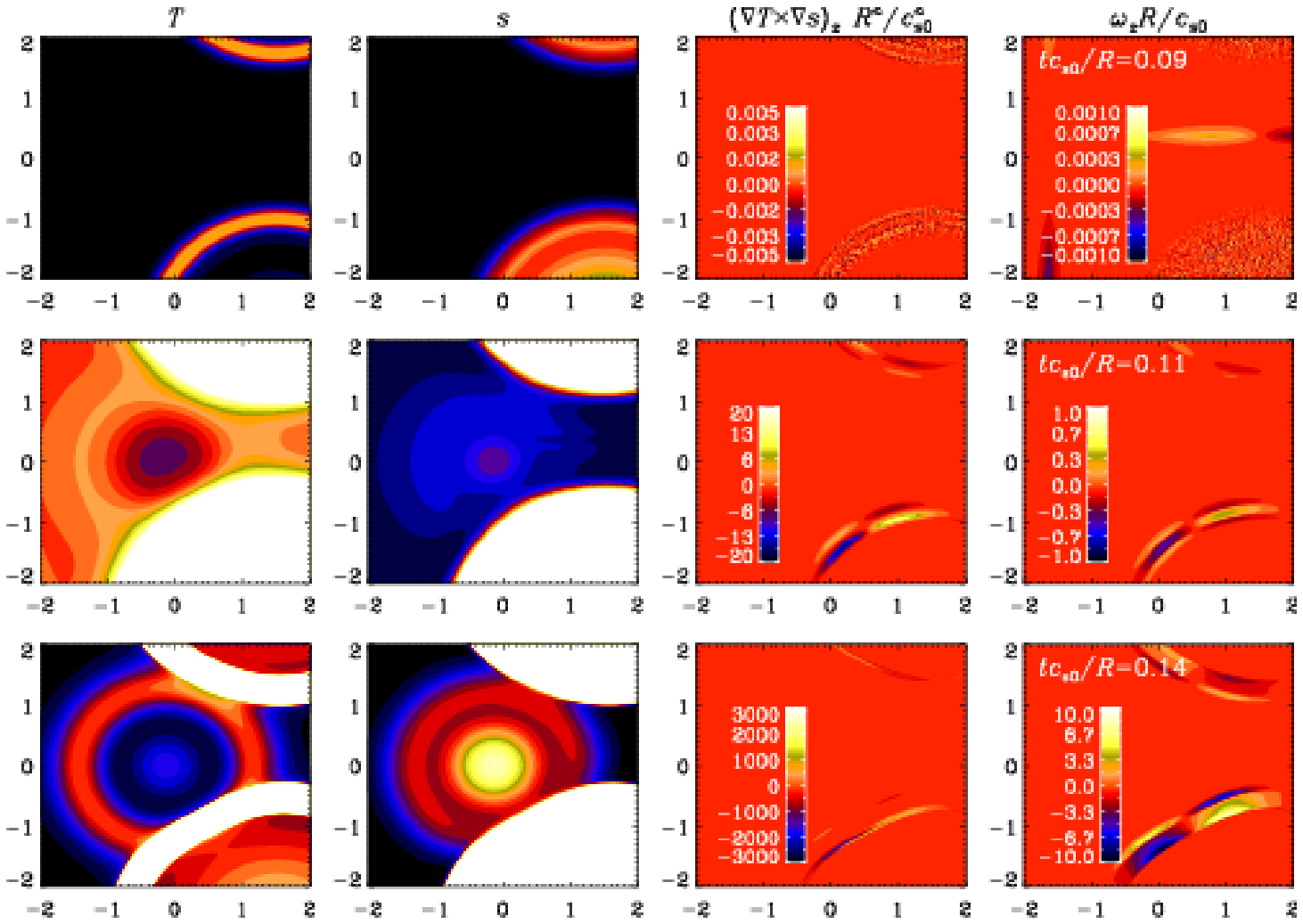



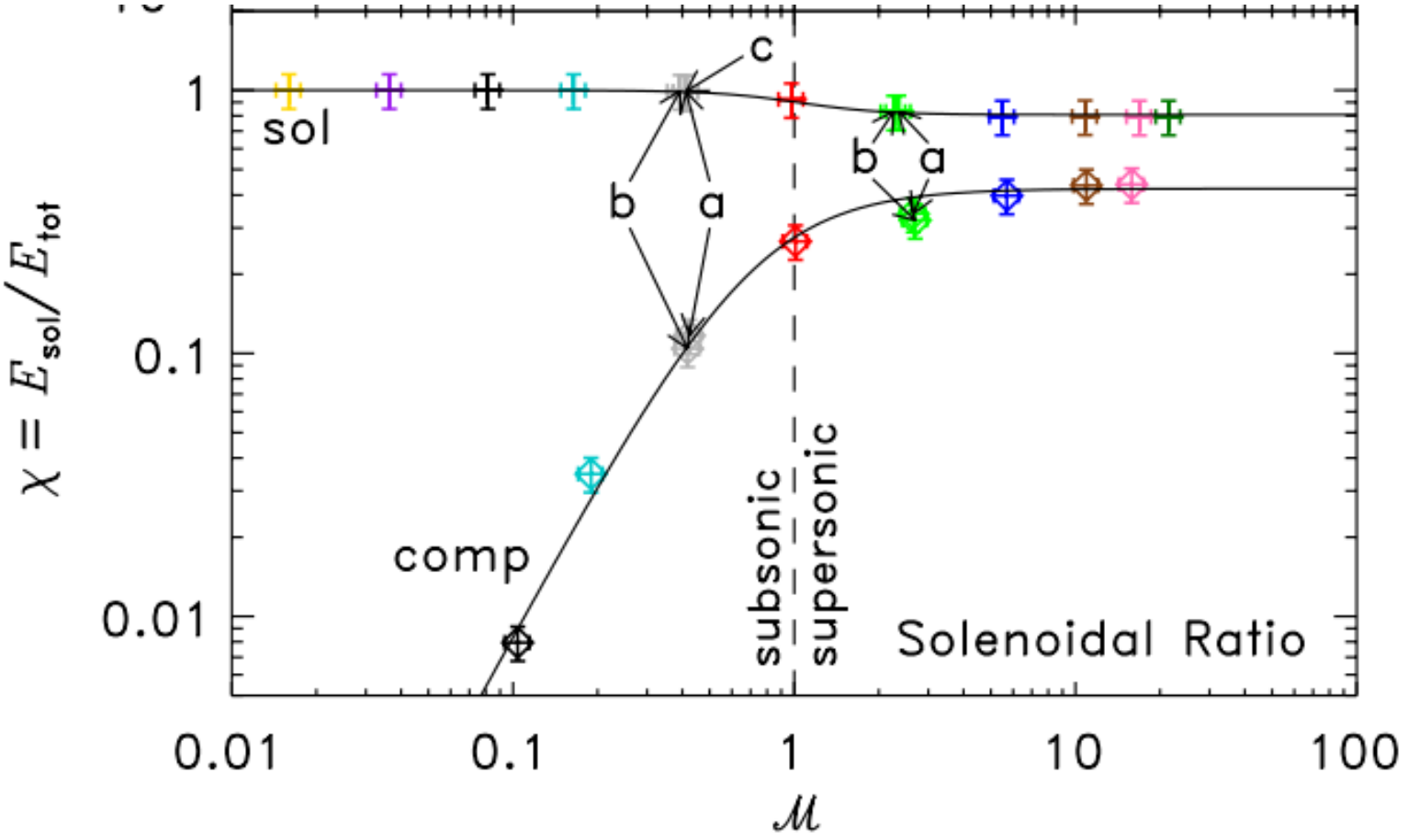
# …after first baroclinic encounter



# $\omega_{rms}$ even in isothermal case

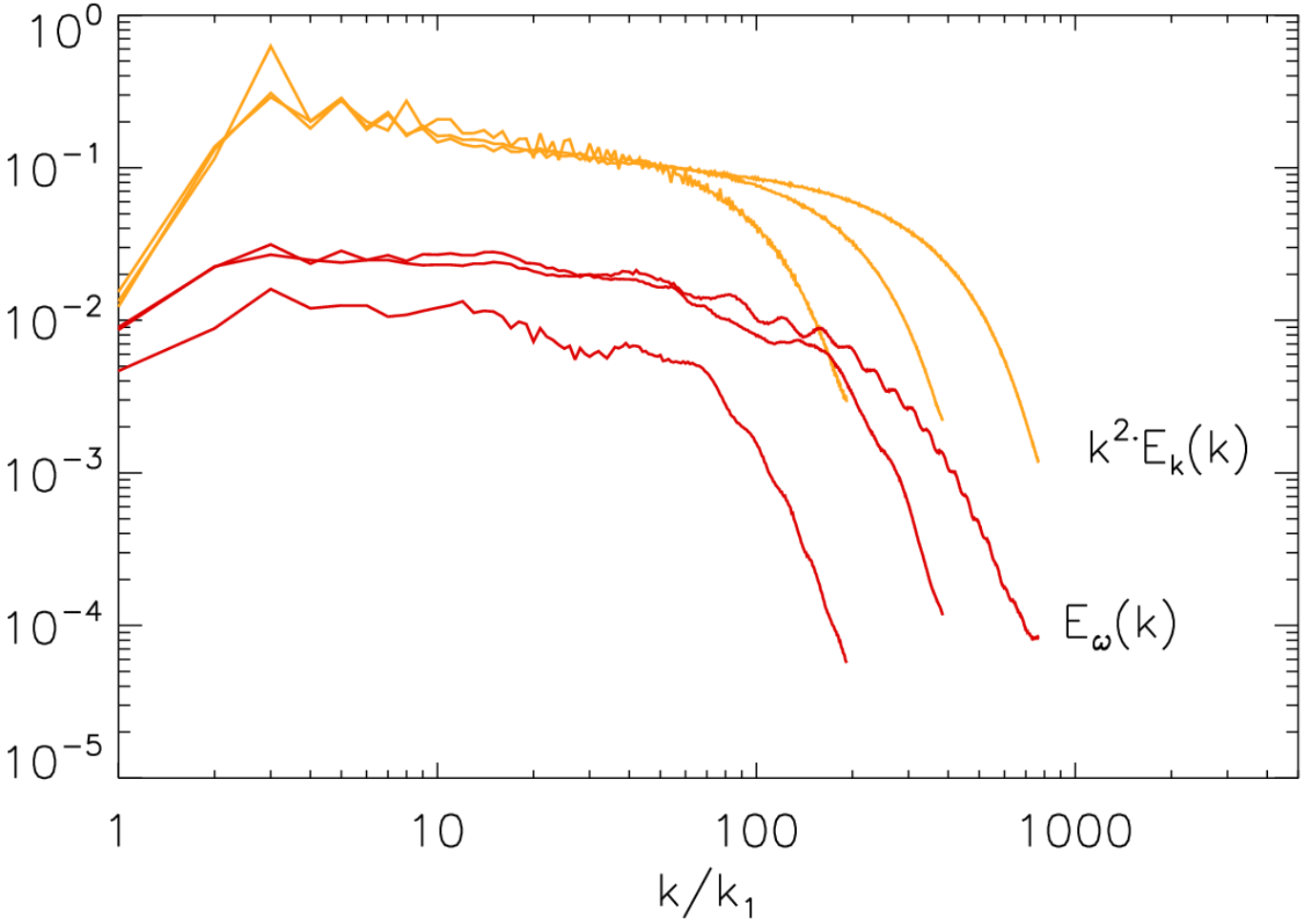
$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times (2\nu \mathbf{S} \cdot \nabla \ln \rho)$$

Federrath et al (1011)



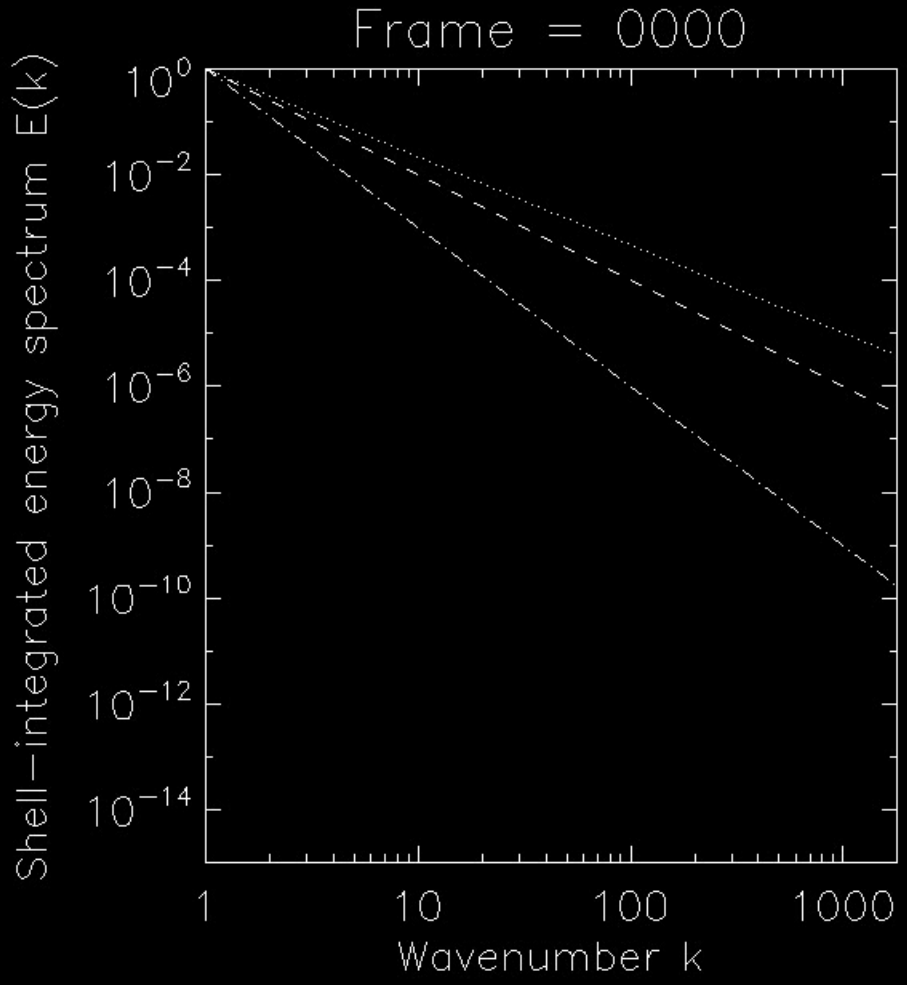
# Spectra with NIRVANA

well behaved at small scale




Calculation by Oliver Gressel

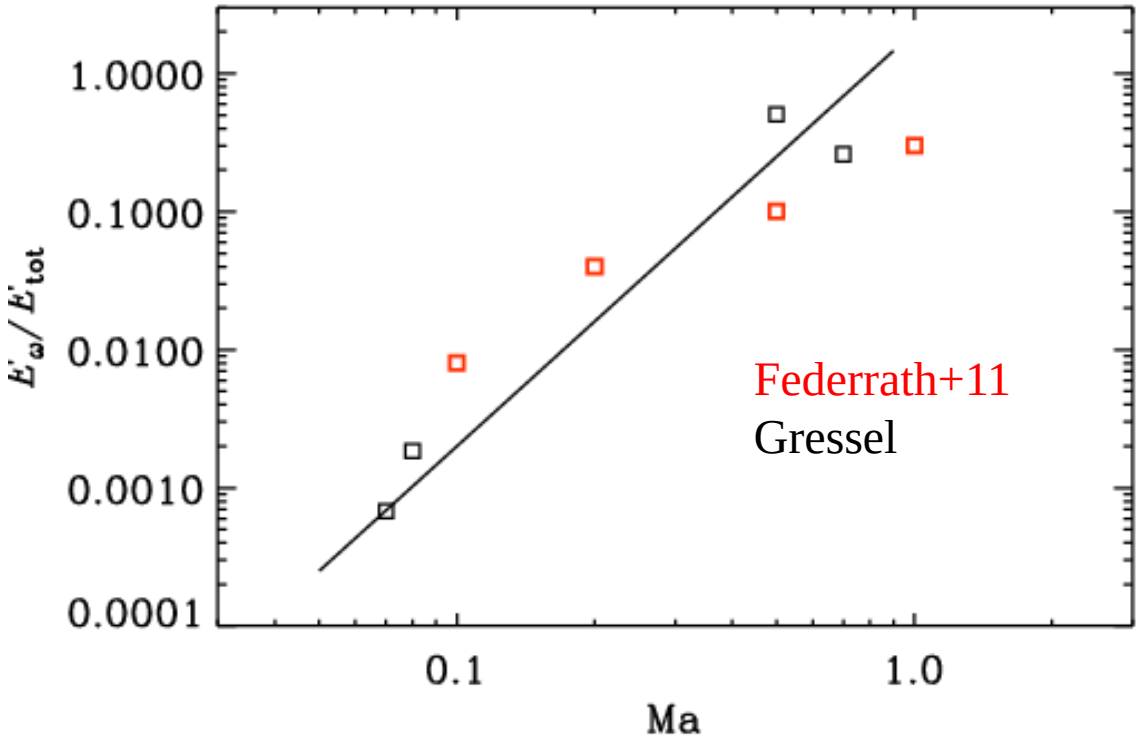
# Calculations by Chi-kwan Chan



# The viscoclinic term

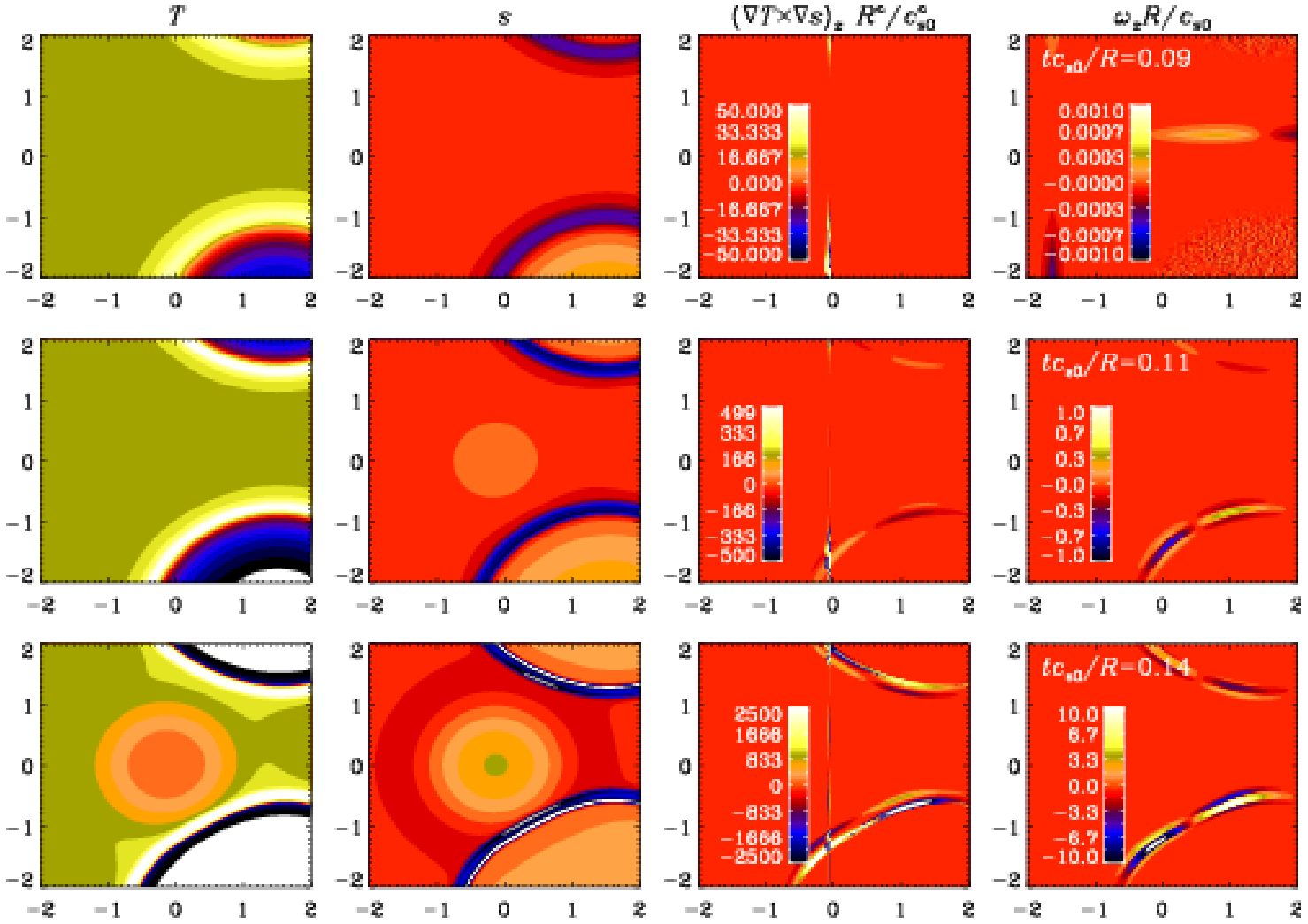
$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) + \nabla \times [\ldots + \nu (\nabla \cdot \mathbf{u}) \nabla \ln \rho]$$
$$= \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) + \nu \nabla \nabla \cdot \mathbf{u} \times \nabla \ln \rho$$

$$\boldsymbol{\omega}_{\mathrm{rms}}^2 \propto Ma^3$$

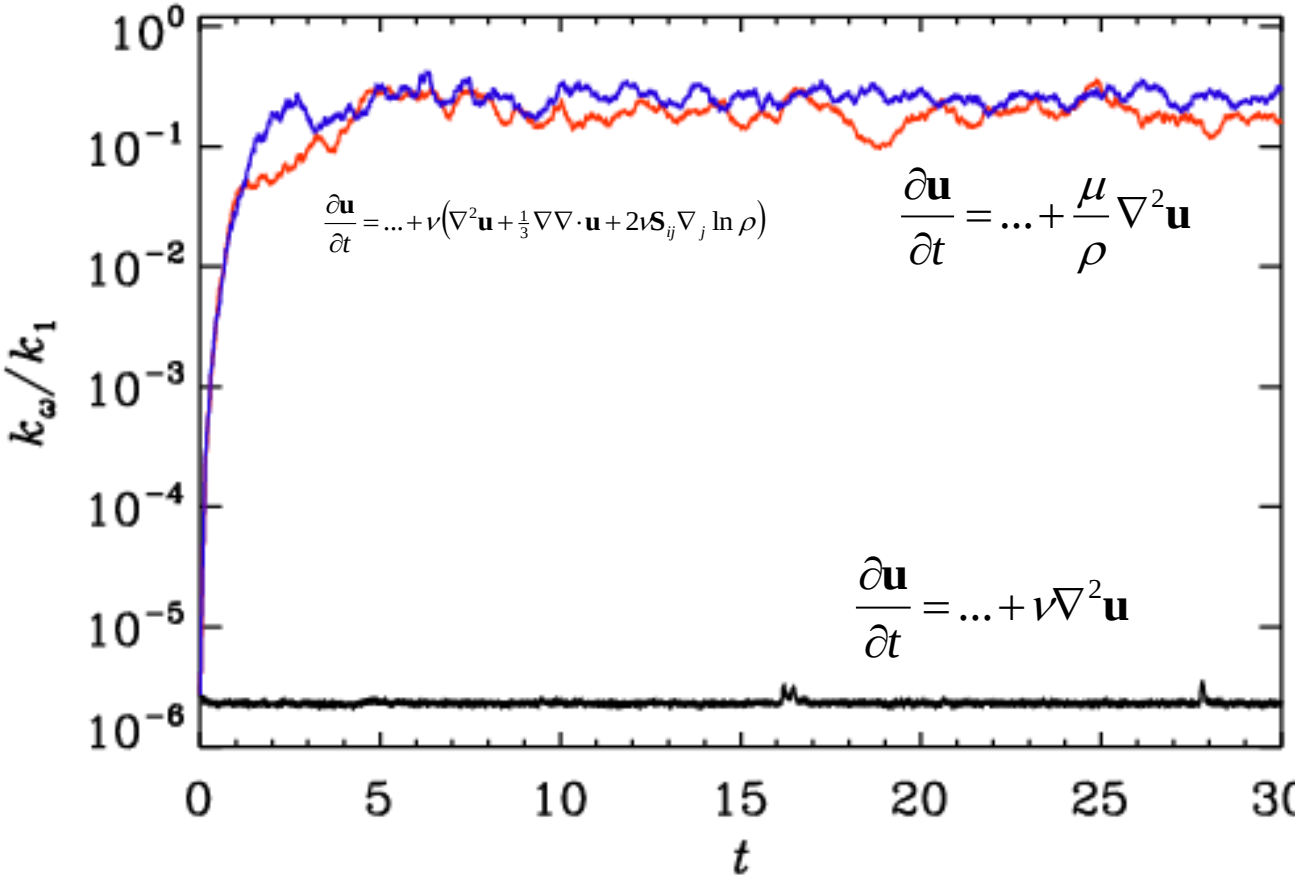


# First viscoclinic encounter

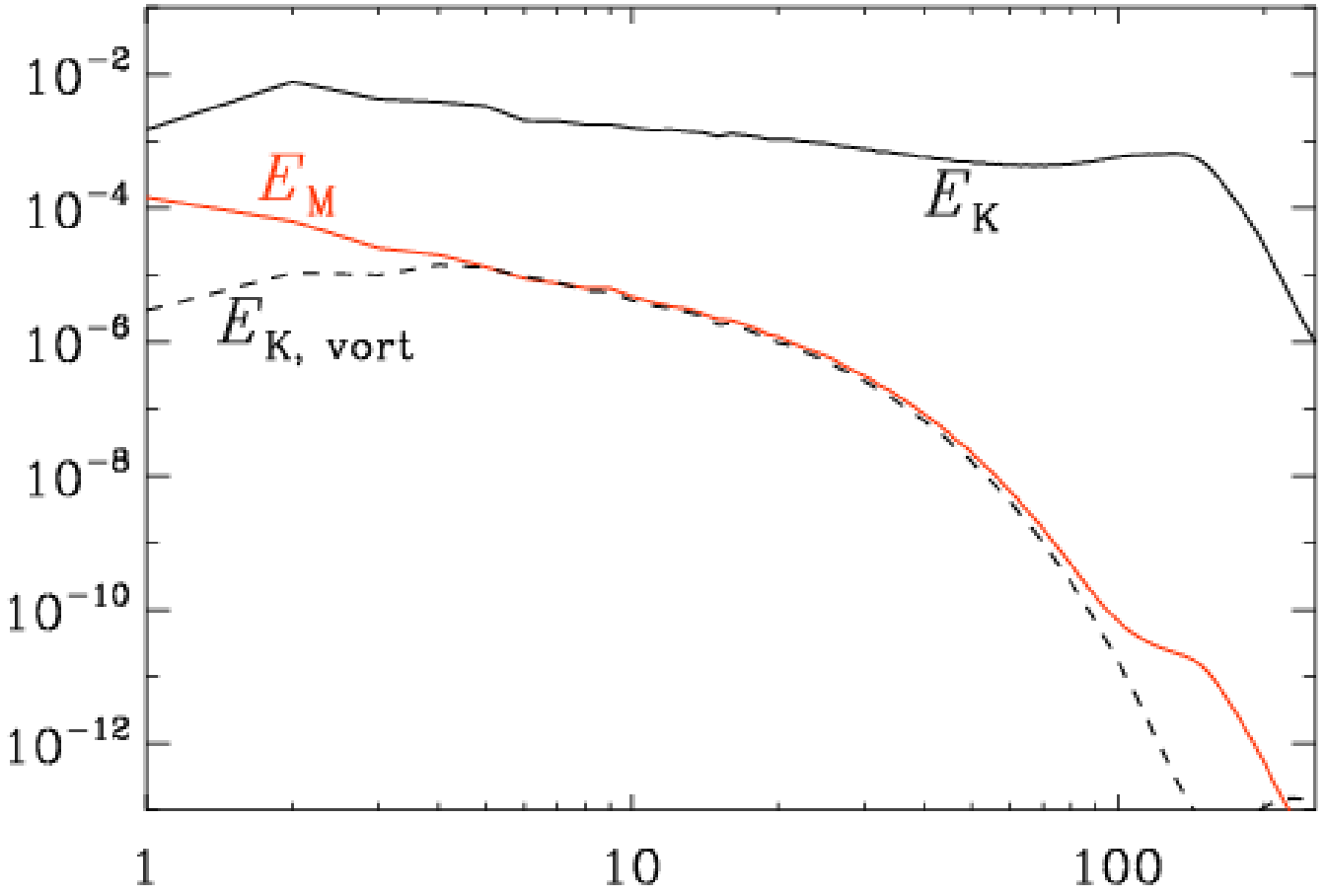
$$\nu\nabla\nabla\cdot\mathbf{u}\times\nabla\ln\rho$$



# Same for μ or ν=const



# B field ignores irrotational part but it also drives vorticity



# Conclusions

- Vorticity unavoidable when compressible
- Numerical effects surprisingly weak!
- With baroclinicity: relative importance unclear
- Magnetic field only due to vortical part