Piecewise quadratic growth during the 2019 novel coronavirus epidemic

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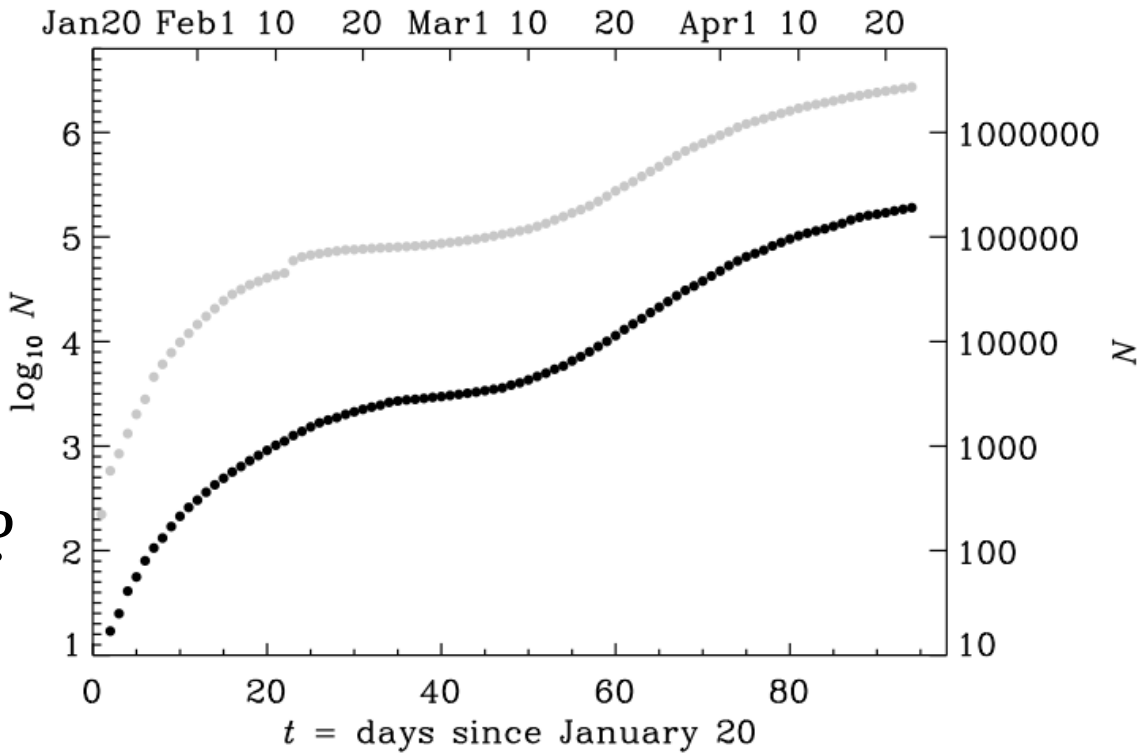
arXiv:2002.03638v2 [q-bio.PE] 14 Feb 2020

- Models predict
  - Early exponential growth
  - Saturation & subsequent decline
- Quadratic growth?
  - Not predicted
  - Generic reason?



Exponential vs quadratic growth

Examples

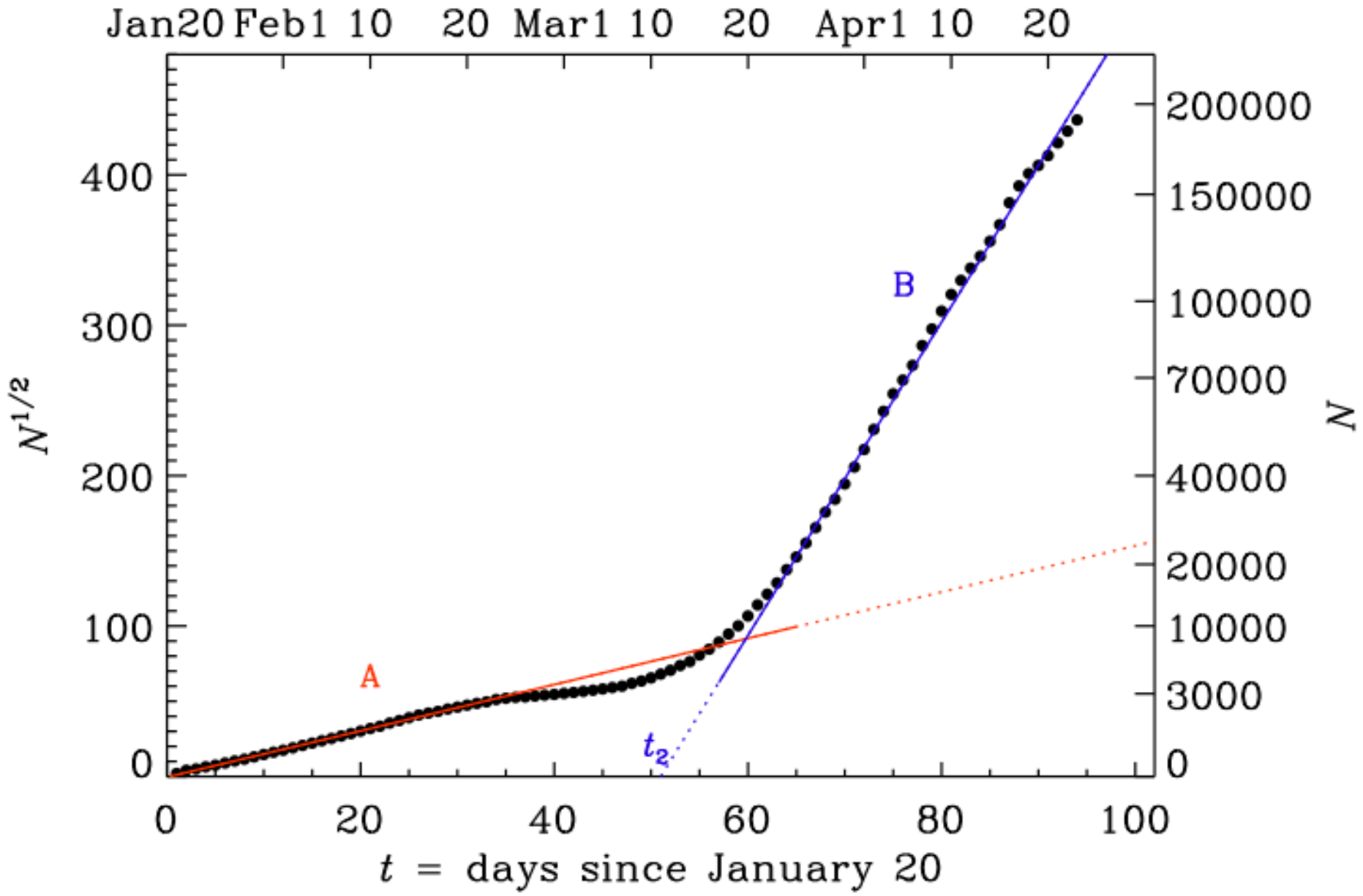
t	0	1	2	3	4	5
2^t	1	2	4	8	16	32
t^2	0	1	4	9	16	25

t	0	1	2	3	4	5	6	7	8	9	10	11
2^t	1	2	4	8	16	32	64	128	256	512	1024	2048
t^2	0	1	4	9	16	25	36	49	64	81	100	121

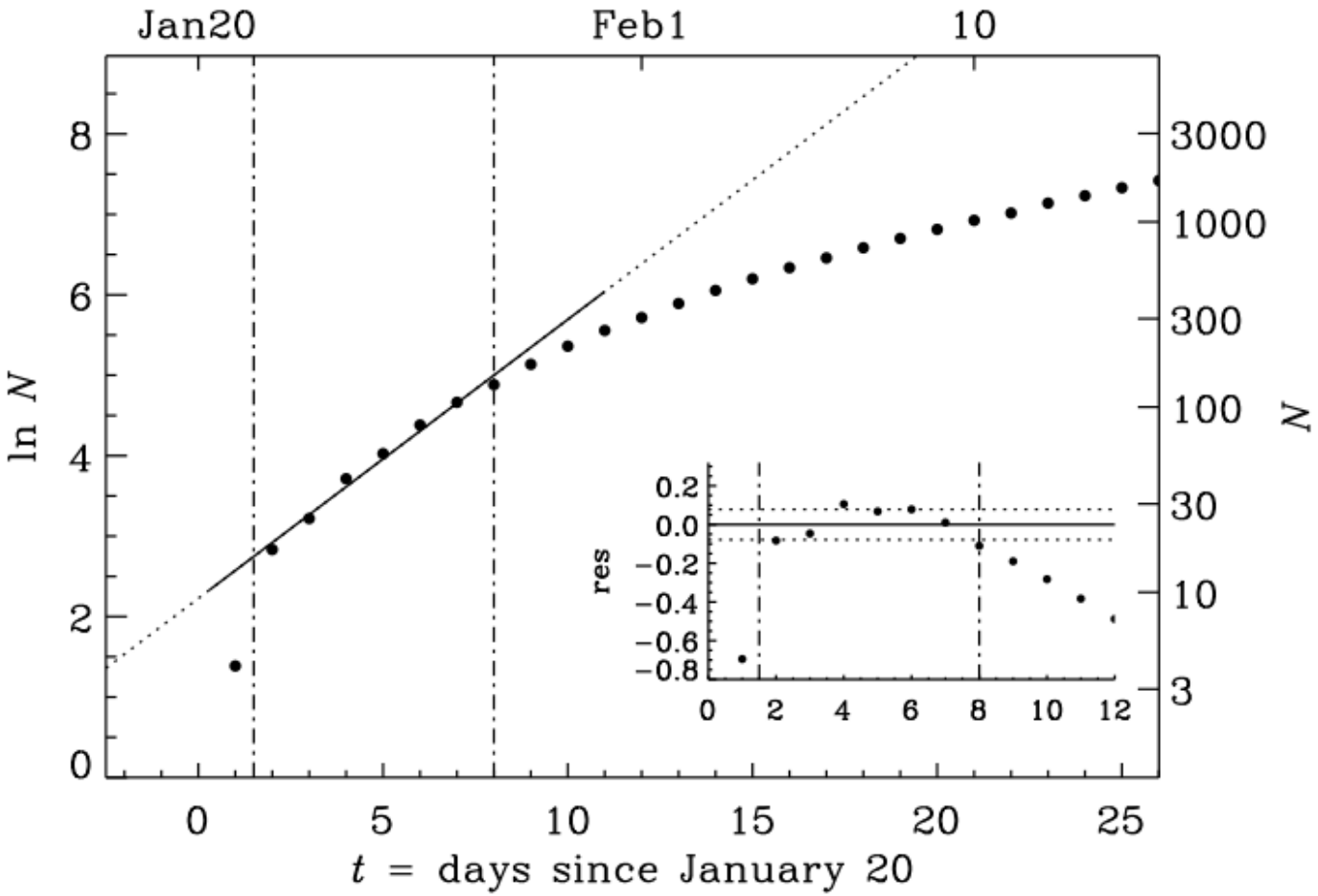
To display:

- Exponential growth? → ln$N$ proportional to t
- Quadratic growth? → $N^{1/2}$ proportional to t

Piecewise quadratic growth



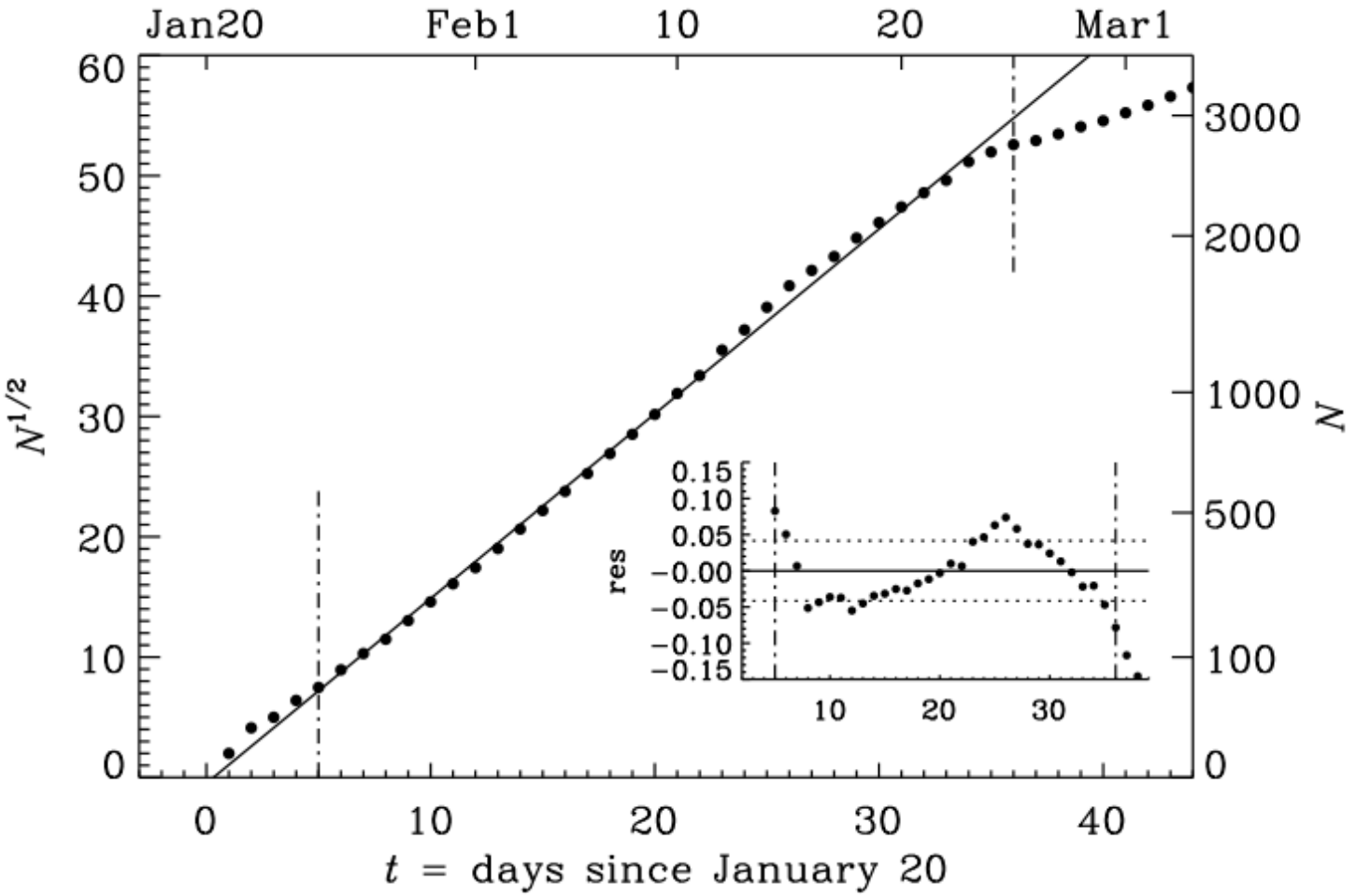
Early exponential growth?



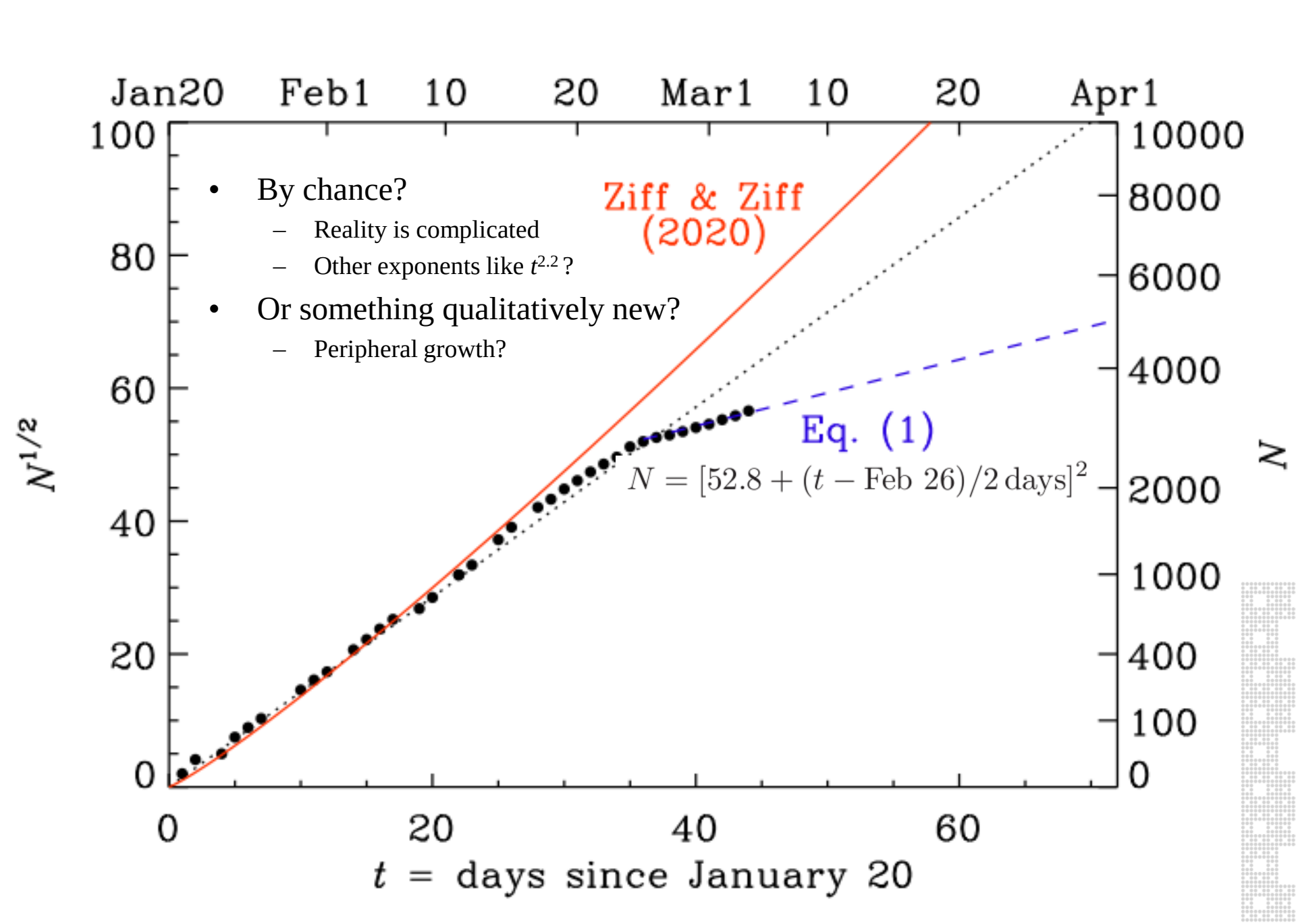
Residual?

$$\mathrm{res} = [N(t)/N_{\mathrm{fit}}(t) - 1] \qquad N_{\mathrm{fit}}(t) = \exp[(t - t_0)/\tau]$$

Comparison: quadratic growth



Residual $\quad \mathrm{res} = [N(t)/N_{\mathrm{fit}}(t) - 1] \qquad N_{\mathrm{fit}}(t) = [(t - t_0)/\tau]^2$


Jan20
Feb1
10
20
Mar1
10
20
Apr1
100
80
60
40
20
0
10000
8000
6000
4000
2000
1000
400
100
0
$N^{1/2}$
N
• By chance?
– Reality is complicated
– Other exponents like $t^{2.2}$?
• Or something qualitatively new?
– Peripheral growth?
Ziff & Ziff (2020)
Eq. (1)
$N = [52.8 + (t - \text{Feb } 26)/2\,\text{days}]^2$
0
20
40
60
t = days since January 20

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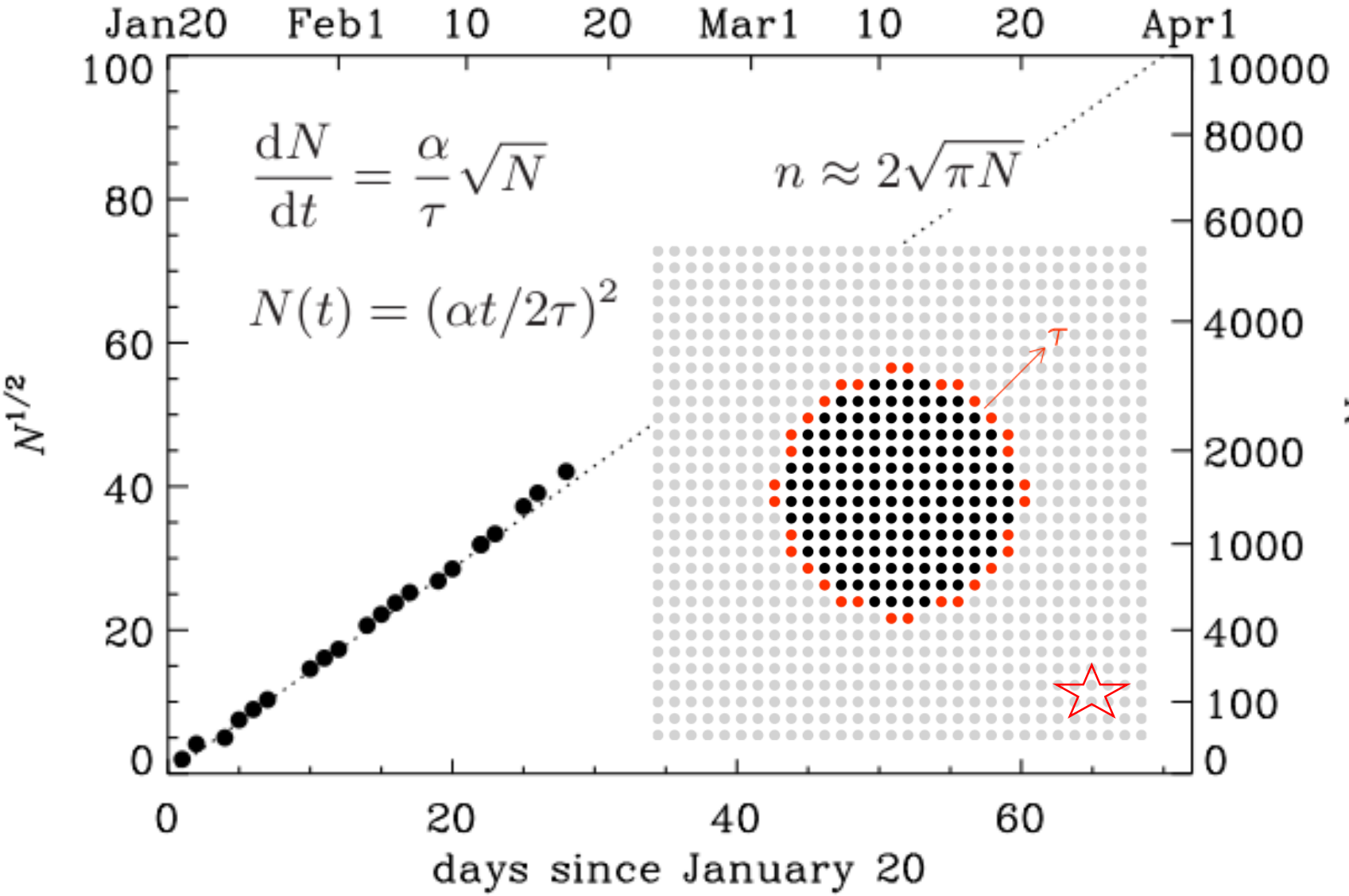
Peripheral growth



SIR model with spatial extent

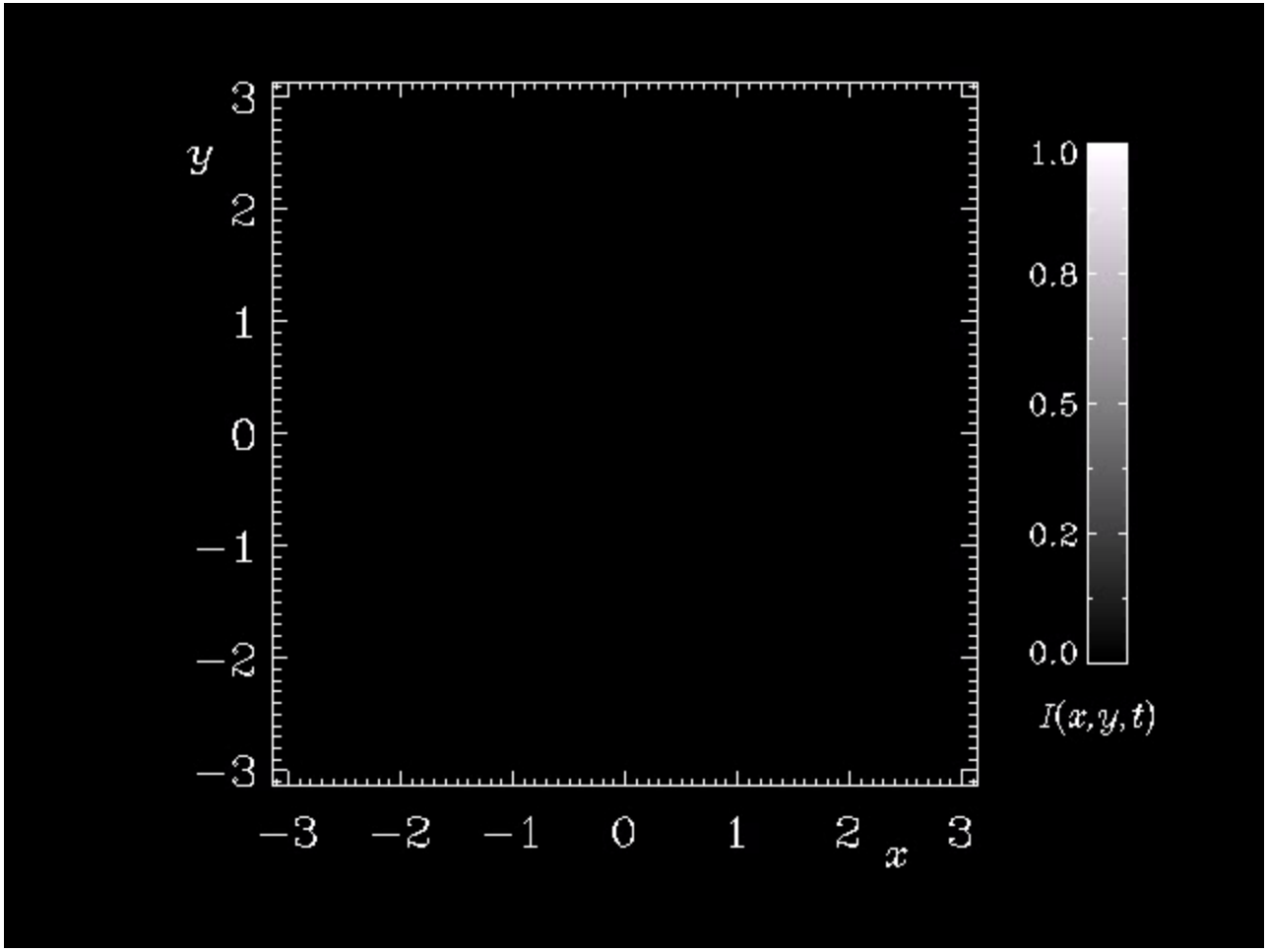
$$\frac{\partial S}{\partial t} = -\lambda S I,$$

$$\frac{\partial I}{\partial t} = \lambda S I - \mu I + \kappa \nabla^2 I,$$

$$\frac{\partial R}{\partial t} = \mu I.$$

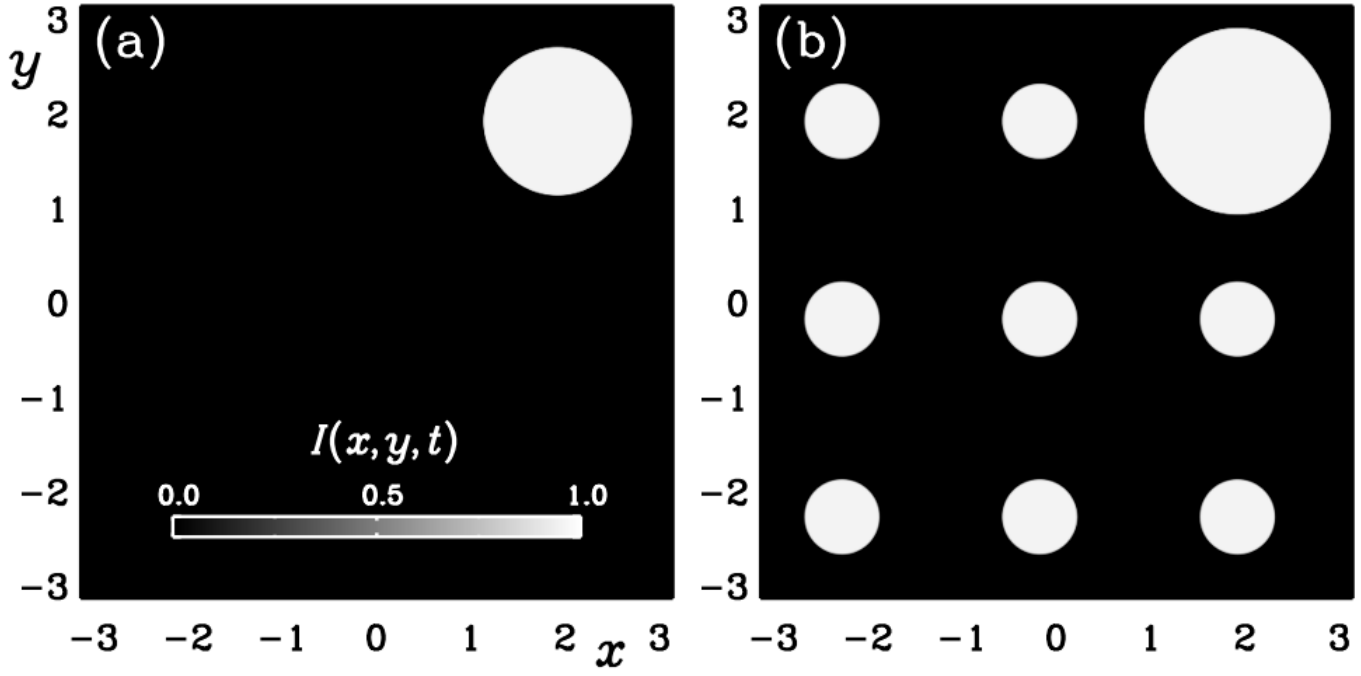
- Usually just exponential growth + saturation
  - Now also spreading (depends on λ and κ)
  - Constant front speed $c = 2\sqrt{\lambda\kappa}$ (Murray, 2003)
  - width $\sqrt{\kappa/\lambda} = 10^{-3}$.

SIR model with spatial extent

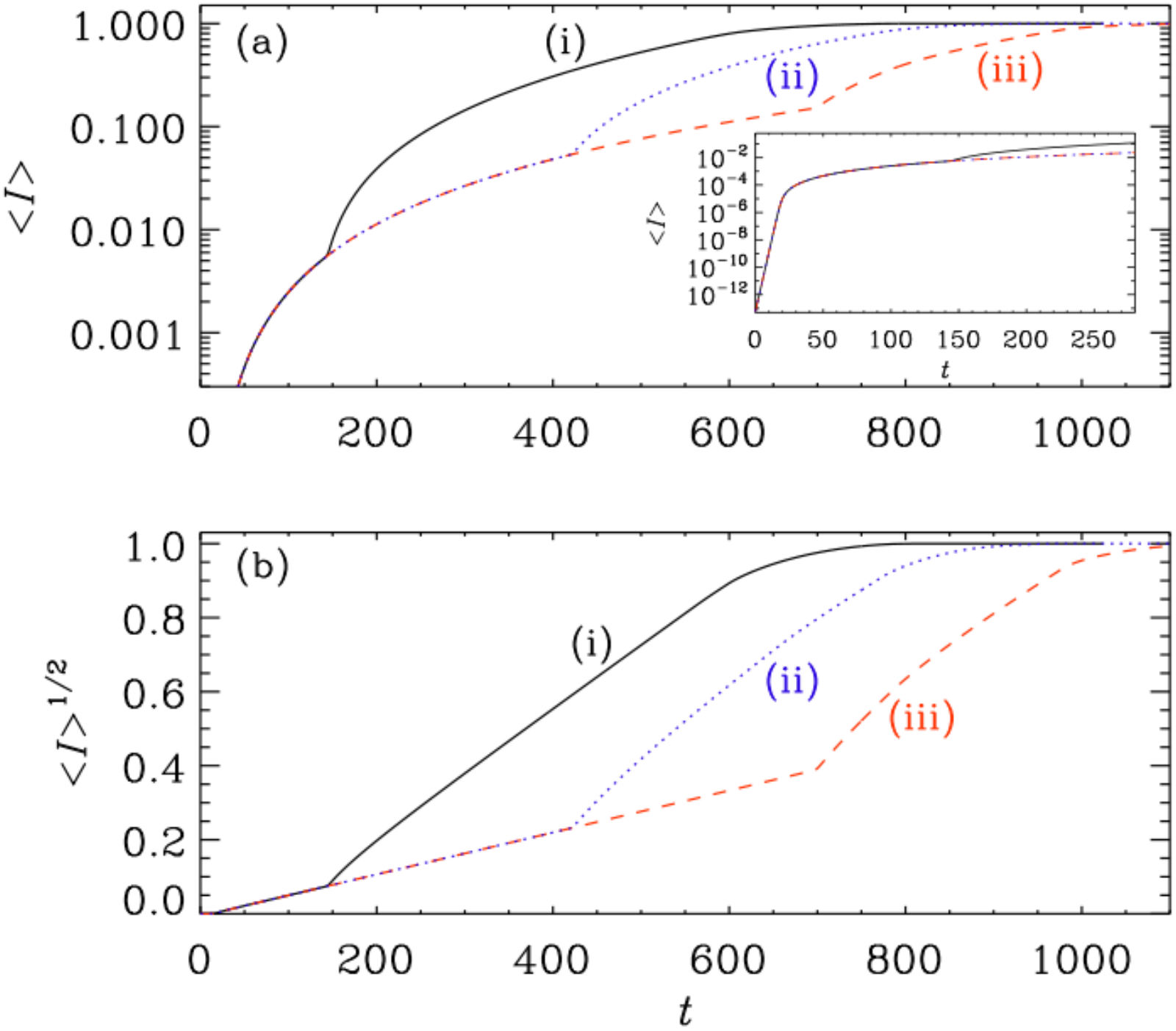


- Growth of one versus several hot spots

SIR model with spatial extent

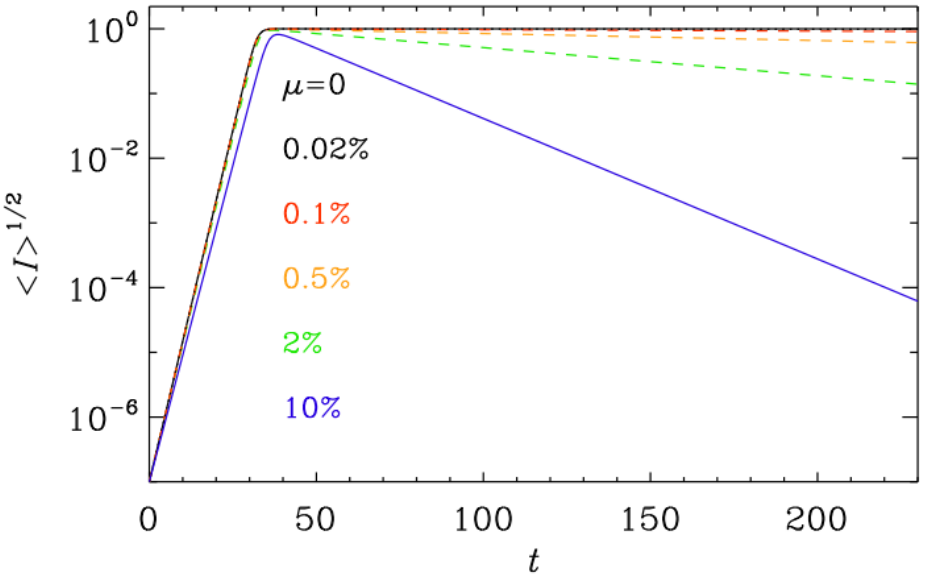


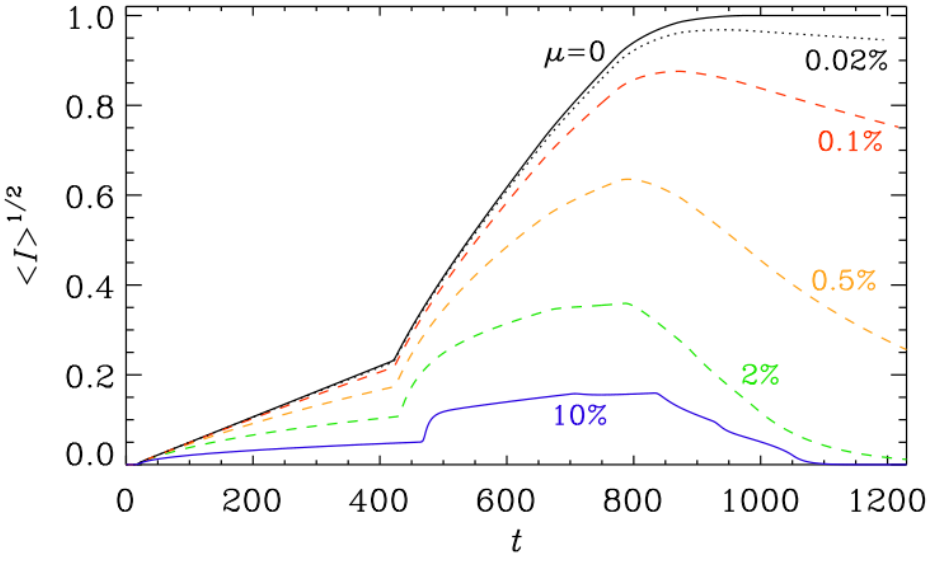
- Other 8 hotspots with much smaller initial *I*.
- Growth faster because of larger surface area



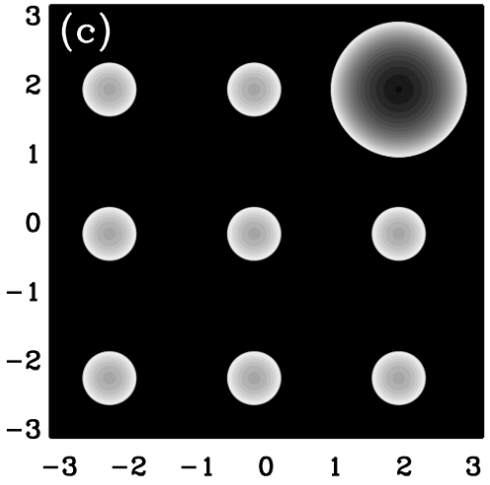
Piecewise quadratic growth perfectly confirmed

Difference: without/with spatial extent

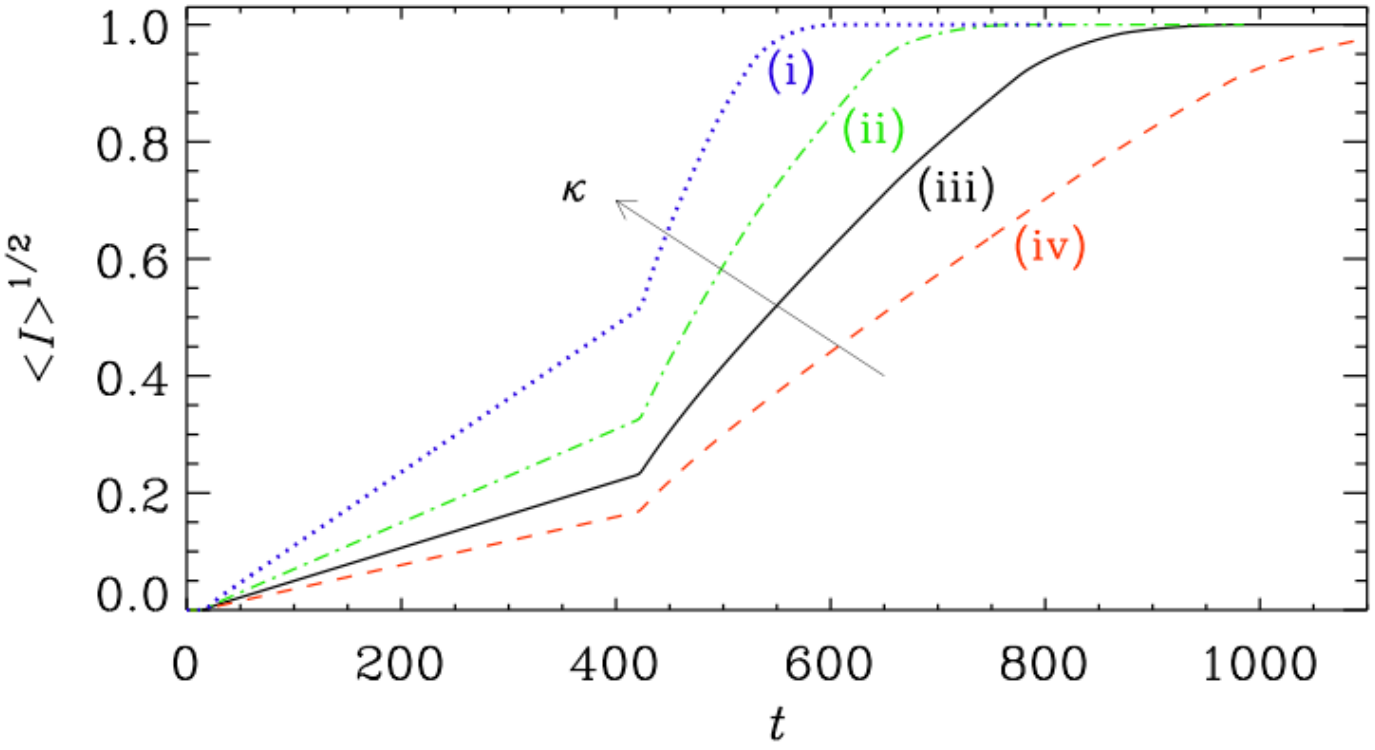




- Saturation very quickly
- Decline in 0D
- But further spreading in 2D



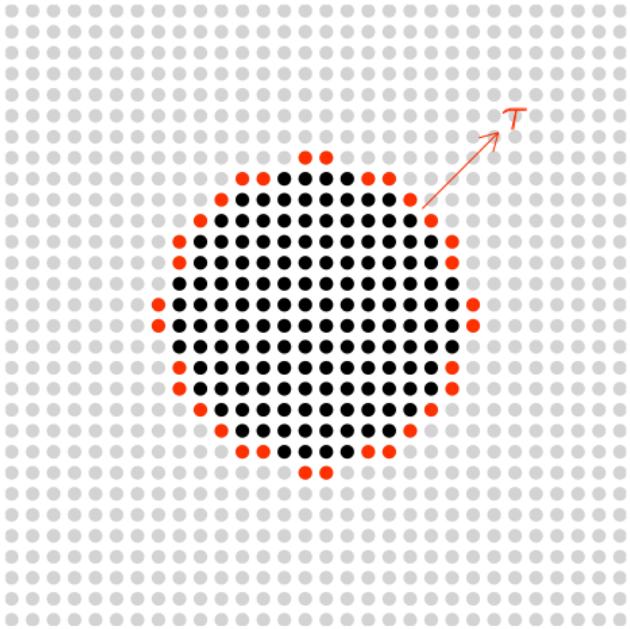
Speed deeps on κ (diffusivity)



- Is an empirically determined quantity
- Comes out as huge → to be checked
- Spatially variable κ → intermediate decline?

Conclusions

- Piecewise quadratic growth observed
  - Explained by peripheral growth
  - Second phase explained several hotspots
- Implications
  - Saturation locally
  - Subsequent spreading
- Can that be possible?
  - Everybody immune?
  - Or everybody isolated?
  - Aerosols??



arxiv.org https://arxiv.org/abs/2002.03638